Comparing models and observations of turbulent clouds

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Summary. It is impossible to deduce the properties of interstellar clouds directly from astronomical observations. Statistical methods are needed to quantify the observational results and to compare them with turbulent cloud models. Different model approaches aiming at an understanding of the cloud physics are introduced. From the large variety of methods designed to quantify different properties of interstellar turbulence and to perform the comparison between models and observations three examples were selected to demonstrate that none of them provides a reliable measure for a particular physical behaviour in all circumstances but that all they have some clear distinctive power to compare models and observations.

1 Observed molecular cloud structure

Observations of molecular clouds show a large variety of complex structures often described as filamentary, irregular or fractal. The confusion in these attributes already shows that there is no agreement on a unique description of the structure neither on the quantification of the structural properties. Thus reproducable and robust methods to characterize complex structures are essential as a first step for the understanding of interstellar cloud physics.

One of the properties often recognized in maps of different tracers of the interstellar medium is their self-similarity, i.e. scale invariance with respect to the actual resolution of the observations and the map size. This can be expressed in terms of a fractal dimension but the actual determination of the fractal dimension for maps of noisy data is not very reliable [12] so that more robust methods are needed. A useful approach is the determination of the power spectrum of observed maps. The scale-invariant size range is then expressed by a power law power spectrum: $P(k) \propto k^{-\beta}$

For rectangular maps, the power spectrum can be easily computed by means of a fast fourier transform (FFT) but problems arise when the measured map contains some invalid data or has a geometry which is adapted to the source an not to the needs of the FFT. Other methods have to be used then.

An alternative method is the Δ-variance analysis [20, 1, 15] measuring the relative amount of structure on a given scale by filtering an observed map by a radially symmetric wavelet with a characteristic length scale l and

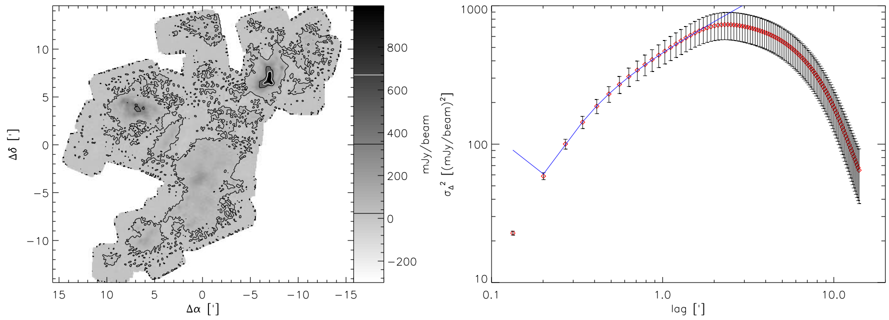

Fig. 1. 1.2 mm dust continuum map of ρ-Oph by Motte et al. [11] and the corresponding Δ-variance spectrum. The solid line shows the best fit with a self-similar scaling taking observational noise and the finite telescope beam into account.

computing the total variance in the convolved map. The spectrum of these Δ-variance values as a function of the filter size l provide a measure for the scaling behaviour of the different components contributing to an observed structure. Self-similar configurations are characterized by a power-law Δ-variance spectrum. Within the typical range of spectral indices measured in interstellar clouds the exponent of the Δ-variance spectrum can be related to the exponent of the power spectrum by $\alpha = \beta - 2$. Compared to the power spectrum the Δ-variance analysis has the big advantage that it can be performed as well for irregular maps or observations with a variable noise across the map. Fig. 1 shows such an example where the Δ-variance analysis was applied to the continum observations of ρ Oph by Motte et al. [11]. On scales not dominated by star formation we find for all molecular clouds a significant self-similar range with $\beta = 2.5 \ldots 3.4$ [1] but here one can also identify the dominant scales in the spectrum given by the star-forming cores, as measured by Motte et al. [11].

2 Cloud modelling

To learn from the observed cloud structure one has to create physical models of the ISM explaining all observed properties. These models should give answers to the questions:

- How were the complex cloud structures produced?
- How does the interstellar medium evolve?
- How does star formation restructure the interstellar medium?

2.1 Fractal models

A first class of models is based on the phenomenological description of interstellar clouds as self-similar structures. A three-dimensional fractal is used for the density and velocity structure. Fractional Brownian motion (fBm) structures are frequently used [1, 2, 10]. They are defined by a power law power spectrum with index β and random phases and provide a perfect match to the spatial scale invariance of the observed molecular cloud maps [20]. They may represent fully developed turbulence with arbitrarily large Reynolds numbers.

Unfortunately, they also bear two severe problems. First it is not clear how one can actually learn anything on the physical behaviour of interstellar clouds from them due to their mathematical nature. Moreover, they always show a Gaussian density distribution in contrast to interstellar clouds which cannot have negative densities and show rather a log-normal distribution. Simple exponentiation would destroy their power-law scaling and the definition of a zero-cut is always arbitrary.

2.2 Hydrodynamic and magnetohydrodynamic models

The alternative, physically justified approach to the interstellar cloud modelling is represented by hydrodynamic or magnetohydrodynamic turbulence simulations. With the recent progress in computing power and modelling technique they start to include effects like self-gravity, various MHD modes, ambipolar diffusion, local heating and cooling, and various dissipation processes over a range of scales from the size of giant molecular clouds down to the size of protostellar cores. (see Mac Low, this volume).

With the possibility to switch different physical processes on and off they are an ideal tool to study the interplay of the processes in the structure formation. It turns out that different processes reveal themselves by different slopes in the Δ-variance spectrum so that the detection of deviations from self-similarity is an essential key to understanding the cloud physics. This may answer the questions how energy is redistributed between the different modes and mechanisms and how molecular cloud turbulence is actually driven.

The models, however, also bear two essential disadvantages. The limited numerical resolution and the unavoidable numerical viscosity limit the turbulence cascade which can be represented today to Reynolds numbers below 10^4, although the interstellar medium is characterised by Reynolds numbers above 10^7. Moreover, the definition of the boundary conditions is always a problem. With either periodic boundaries or a fixed finite box they can only represent infinitely large or very small configurations.

2.3 The radiative transfer problem

With the given density and velocity structure from the models it is still not straight-forward to compute the observable properties of the clouds, like molecular line maps or profiles. Here, the full complexity of the radiative transfer problem enters. Translating densities, velocities, temperatures and abundances into the observable line/continuum intensities requires a considerable effort in computing power or sophisticated approximations.

Systematic investigations by Padoan et al. [17] and Mac Low & Ossenkopf [8, 13] have shown, that there is in general no fixed relation between the column density and any line intensity but that the Δ-variance spectra of maps from low density tracers still provide a good measure for the large-scale scaling behaviour. They show, however, that line maps from fractal models remain self-similar in all transitions in contrast to observations which show an increasing confinement of the emission when going to energetically higher molecular transitions.

3 Methods to compare observed and simulated data

Because the models do not generate an exact representation of the observed interstellar clouds the comparison of the spatial and the velocity structure between model results and observations has to be based on statistical measures. A relatively large set of independent parameters has been proposed in the literature. Characterising either the intensity distribution, the isotropic scaling behaviour, the structure in velocity space or the anisotropy of the structure, different methods have to be combined to allow a relatively complete comparison between models and observations. For a general overview see [12]. Unfortunately, our current knowledge on the cloud physics is still insufficient to determine a clear relation between a particular statistical measure and a physical mechanism, so that we have to combine many measures today to obtain conclusions on the match between a cloud model and observational data. As it is impossible to give even a rough overview over all methods here, I will introduce only a few examples which are currently discussed.

3.1 Channel maps

A picture on the combination of the density scaling and the velocity structure can be obtained when analysing the statistical properties of channel maps. Brüll et al. (this volume) have shown that the Δ-variance spectrum of channel maps in the Galactic Ring region can be used to determine the size or distance of gravitationally collapsed cores. They allow to distinguish inter-arm material from structures in the spiral arms and the Galactic Ring.

The statistical relation between neighbouring channel maps can be quantified by means of the velocity channel analysis (VCA) introduced by Lazarian & Pogosyan [6]. For fractal clouds this method allows to deduce the spectral indices of density and velocity structure as demonstrated in Fig. 2. Theory predicts for the fractal model used here, that the channel index changes from $\beta_{\rm channel} = 4.5 - \beta_v/2 = 2.5$ for narrow channels to $\beta_n = 3.7$ for broad channels. Applying the VCA to an HI map of the SMC and assuming a fractal structure Stanimirović & Lazarian [18] were able to derive the spectral indices of the density and velocity structure simultaneously.

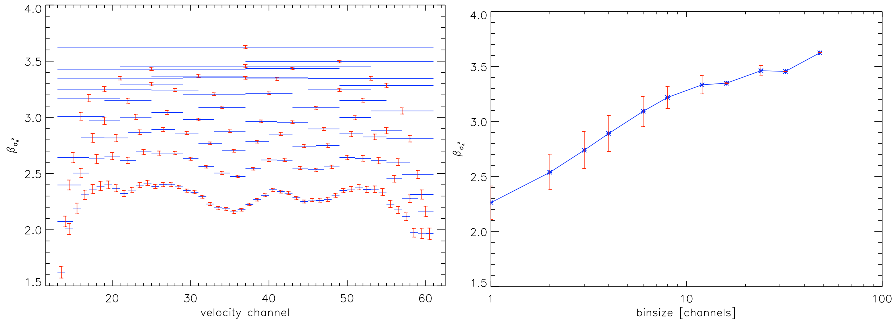

Fig. 2. Spectral indices $\beta_{\rm channel}$ of channel maps with varying width for a fractal cloud model with $\beta_n = 3.7$ and $\beta_v = 4.0$. The right panel shows the dependence of the average channel index as a function of the channel width.

Unfortunately, the VCA is only justified if the data represent a completely sampled fractal structure [4]. In contrast, molecular cloud observations often show indications of low-number statistics or intermittency, visible as non-Gaussian average line profiles. Numerical experiments using hydrodynamic simulations with a self-similar behaviour over a long but limited range of scales have proven that the VCA is no longer able to derive the spectral indices there. Thus the applicability of the method has to be tested for every set of observational data.

3.2 Centroid velocity maps

Centroids as first velocity moments are much more robust to observational effects than single channel maps so that they are often considered to be a reliable measure for the velocity structure [9]. The Δ-variance spectra of nested centroid maps from CO observations in the Polaris Flare covering three orders of magnitude in spatial scales [15] showed a contineous slope change from $\beta_{\rm centroid} = 2.8$ at the largest scale to $\beta_{\rm centroid} = 3.2$ at small scales. From the comparison with the results obtained in MHD computations Ossenkopf & Mac Low [14] concluded that they represent a spectral index of the underlying velocity structure which is larger by one.

In contrast, Miville-Deschênes et al. [10] found that the spectral index measured in centroid velocity maps from fBm models agrees with the spectral index of the used velocity fBm. This contradiction was noticed by Brunt & Mac Low [3], but not fully explained yet. The actual explanation of the problem arises from the variable role of projection smoothing depending on the ratio between the average density and the density variation. This is demonstrated in Fig. 3 illustrating the role of the required density shift in fBm models. As discovered by Lazarian & Esquivel [7], the centroid map can be either density or velocity dominated. The figure shows that this transition

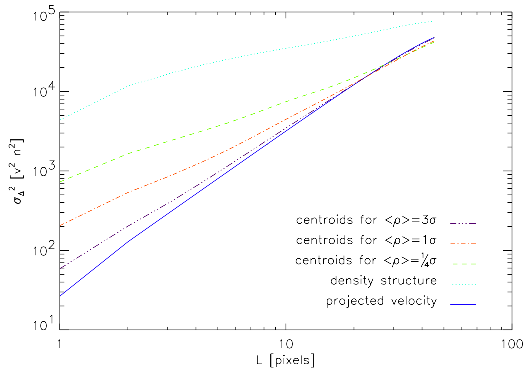

Fig. 3. Δ-variance spectra of centroid velocity maps obtained from the same fBm model when varying the average density. For comparison we have also shown the spectra of the pure density and the projected velocity structure multiplied by the variance of the other quantity to obtain the same units.

from centroids reflecting the density structure to centroids reflecting the velocity structure is automatically obtained when reducing the density variation relative to the average density. In most cases we will, however find a mixture. Without an independent measure for this ratio it is impossible to distinguish both behaviours. Thus, a centroid map alone does not allow to derive any spectral index.

3.3 Clump decomposition

Another method which is frequently applied to identify the combined density-velocity structure is the decomposition of molecular line maps into separate clumps. Various methods exists for this purpose [19, 21, 16], resulting in slightly deviating but comparable spectra of clump properties. To test the significance of the clump decomposition we have applied GAUSSCLUMPS [19] to MHD simulations. For an example model Fig. 4 displays the number of clumps as a function of their mass when determined either in the original density structure or in simulated molecular line observations of the same structure. The straight line indicates the slope which is observed in most CO observations [5]. The observed clump spectrum hardly reflects the physical distribution of clumps. When comparing individual clumps only the most massive clumps are identically indentified in both data types. Moreover, the result confirms the effect of a pseudo-virialisation of the clumps in the molecular line data due to radiative transfer effects as discussed by Mac Low (this volume).

An interesting result of the clump decomposition is, however, that none of the investigated cloud models exactly reproduces the exponent measured

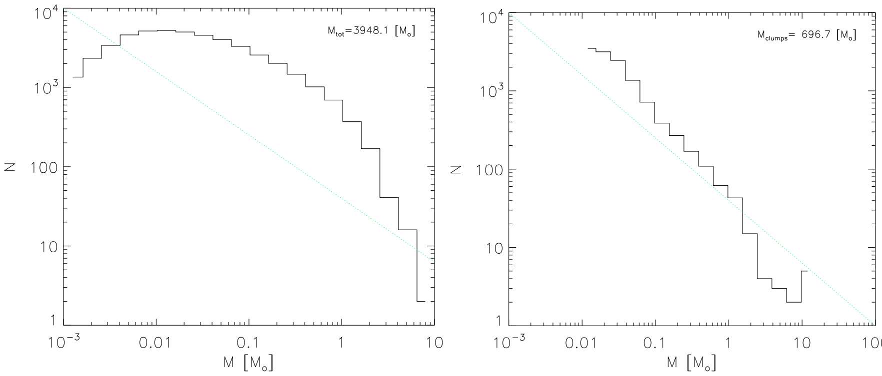

Fig. 4. Clump mass spectra computed from the original density structure and the computed ^{13}CO 1-0 map simulated with a S/N=20.

in most observationally based spectra when computed with the same signal-to-noise ratio. The clump spectrum is able to distinguish between different models so that it is still a valuable structure parameters even if we currently do not understand its origin.

4 Summary

4.1 Results of the comparison between models and observations

None of the studied models fit all observational constraints. Fractal models show a wrong mutual relation between map statistics obtained for different tracers and either a wrong density statistics or wrong scaling laws. (Magneto-)hydrodynamic models fit a too small inertial range and provide wrong spectral indices in clump statistics and velocity channel analysis.

Nevertheless, the comparison between models and observations already allows to constrain some essential parameters governing the structure of molecular clouds. The main turbulent energy injection must occur on very large scales. Strong magnetic fields are excluded and a turbulent driving may have stopped only recently. The deviations from a self-similar scaling behaviour in star-forming regions reveals the role of self-gravity and we find first indications for the dissipation by ambipolar diffusion. The actual understanding of the molecular cloud physics is always hidden in the observation of deviations from self-similarity.

4.2 Conclusions

An iterative process of constructing cloud models, comparing their appearance in observable tracers with actual measurements, and improving the models to resemble the observational results is required to learn about the nature of interstellar turbulence. All methods for the comparison investigated so far show some distinctive power for the comparison between models and observations. However, they almost never show what they are thought to show. Currently we still do not know which method reflects a particular physical behaviour best. The use of any single method to derive cloud physics will almost necessarily fail. Beside the ongoing comparison between models and observations new classes of tools are needed to characterize e.g. the anisotropy of the structures or being sensitive to the velocity-density correlations in the clouds.

New cloud models have to provide a larger dynamic range, a self-consistent treatment of the energy balance and the the chemical structure of the clouds. Continuing this approach will help to recover the nature of interstellar turbulence finally providing a self-consistent picture for the evolution of interstellar clouds including the complex of star-formation.

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