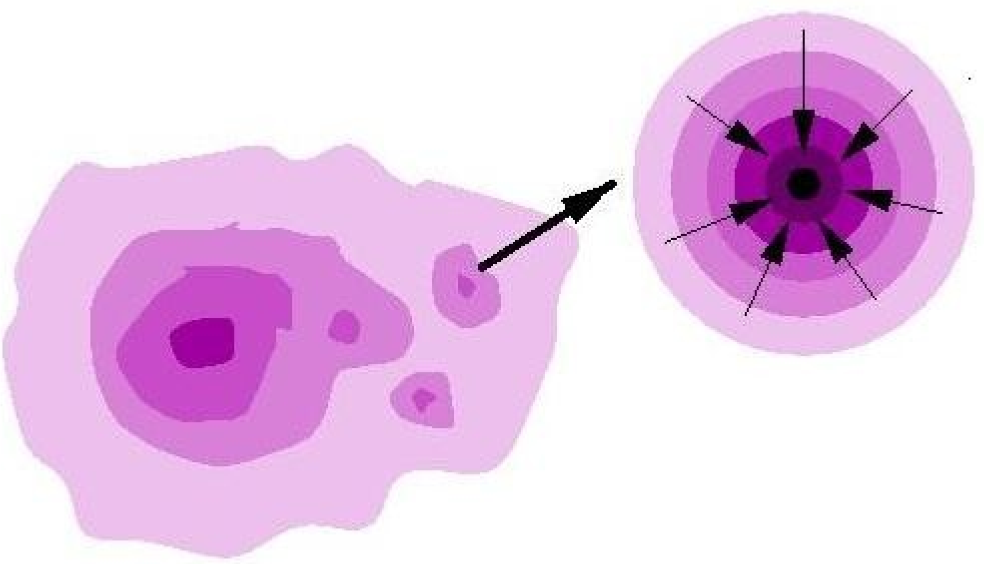
Star formation

Protostellar Cores, Stability and Collapse

Starting point

- Dense cores on molecular clouds



- Do they collapse?

Isothermal cloud in pressure equilibrium

1-D equilibrium between gravity and pressure:

Pressure from gravity:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

Included mass:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

which can be combined into the Emden equation

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

Isothermal cloud in pressure equilibrium

Combine with equation of state (Bernoulli equation):

$$P = n\,k\,T = \frac{k\,T}{m}\,\rho = c_s^2\,\rho$$

with the isothermal sound speed

$$c_s = \sqrt{\frac{\partial P}{\partial \rho}} = \sqrt{\frac{kT}{m}} \approx 0.06\sqrt{\frac{T[K]}{m/m_{H_2}}}\,[\mathrm{km\ s^{-1}}]$$

m is the mass of a gas particle

Singular isothermal sphere

Shu 1969, 1977:

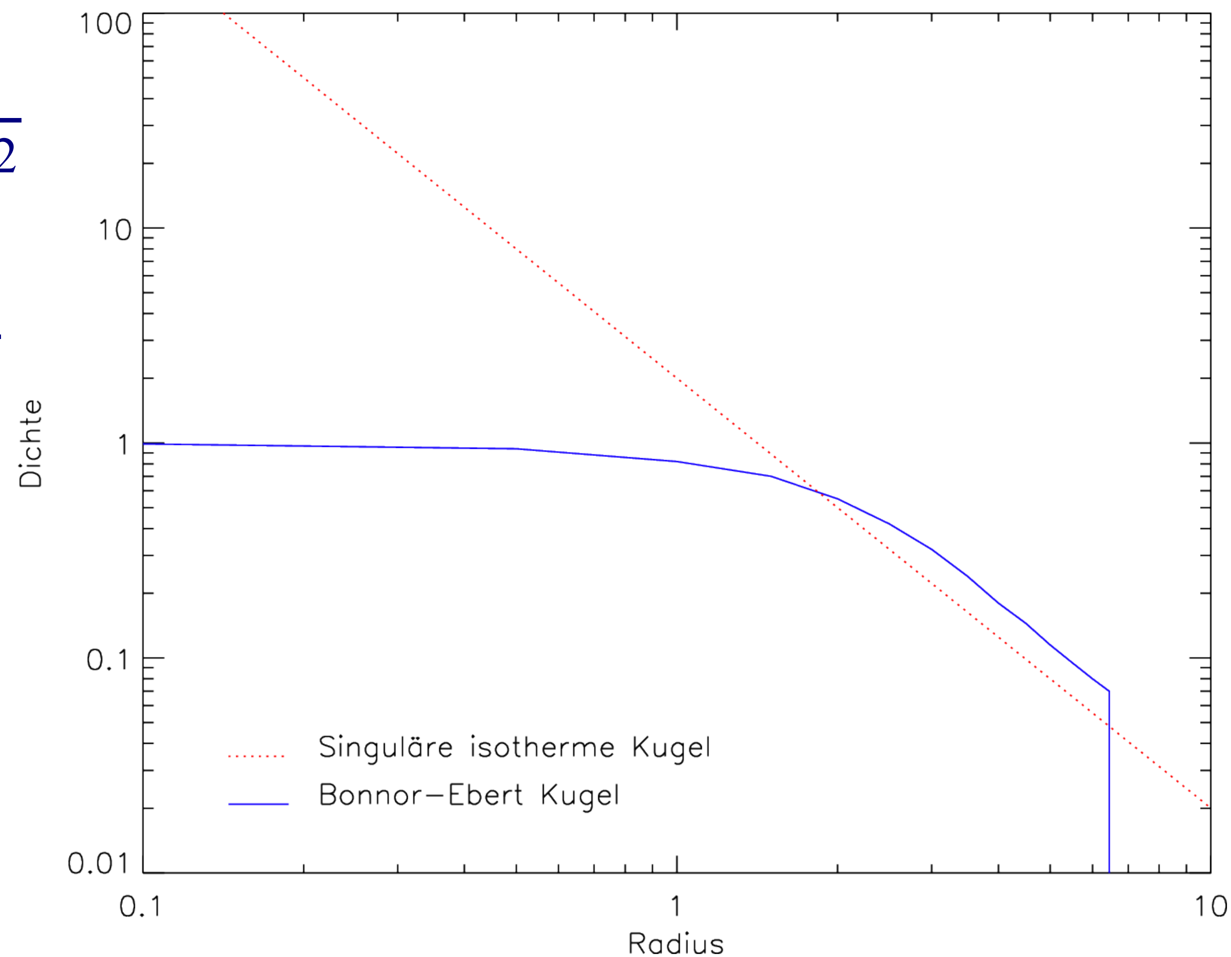
Ignore singularity at $r=0$

Density and mass distribution

$$\rho(r)=\frac{c_s^2}{2\pi G r^2}$$

$$M(r)=2c_s^2\frac{G}{r}$$

= Scale-free solution!



Bonnor-Ebert sphere

solved with the boundary conditions

$$\rho(0)=\rho_c \text{ and } \left.\frac{d\rho}{dr}\right|_{r=0}=0$$

using variable substitutions

$$y=\frac{\rho}{\rho_c};\; x=r\sqrt{\frac{4\pi G m \rho_c}{kT}}$$

leads to the following form of the Emden equation

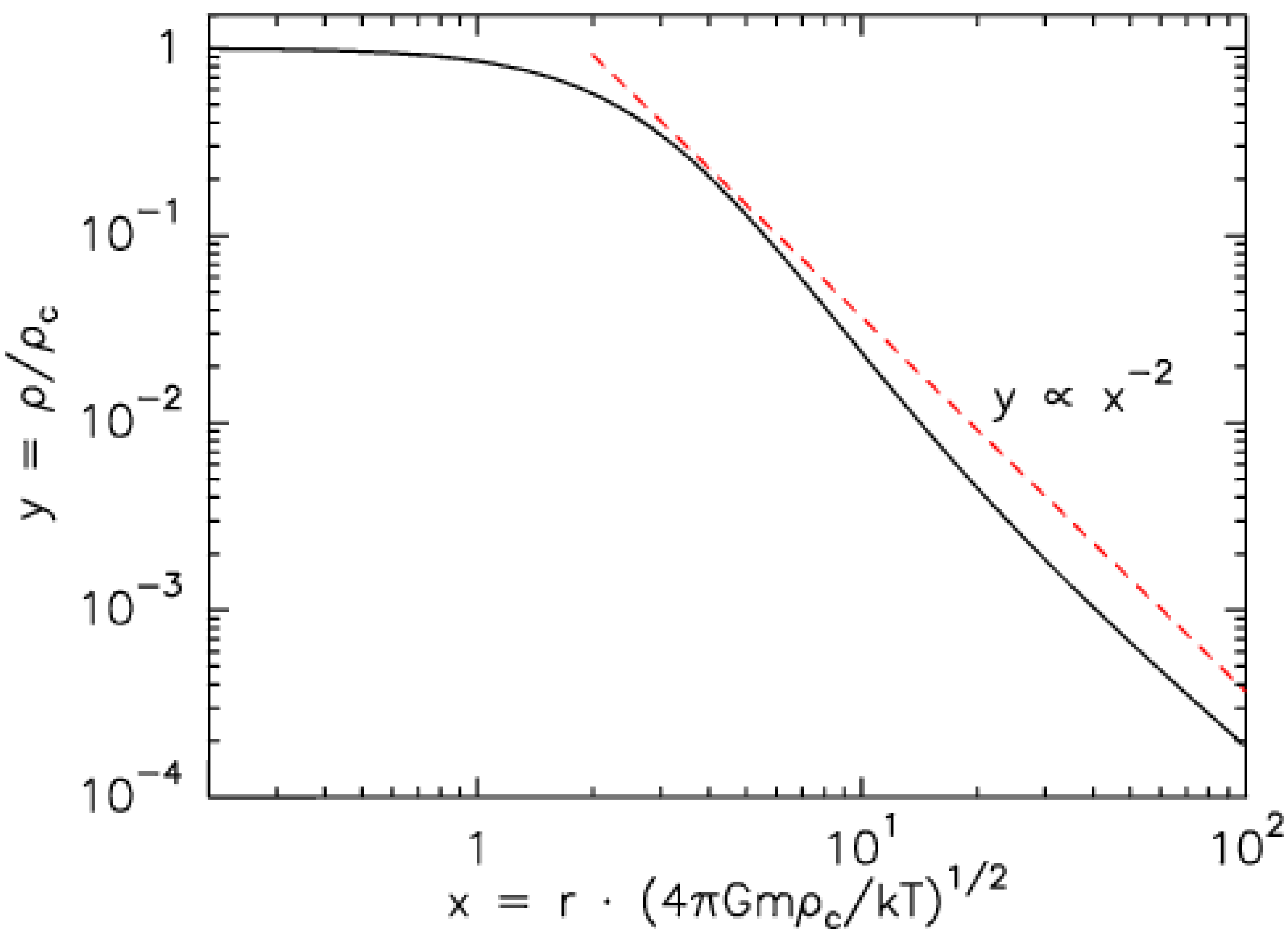
$$y''-\frac{y'^2}{y}+\frac{2y'}{x}+y^2=0$$

with boundary conditions

$$y(0)=1 \text{ and } y'(0)=0$$

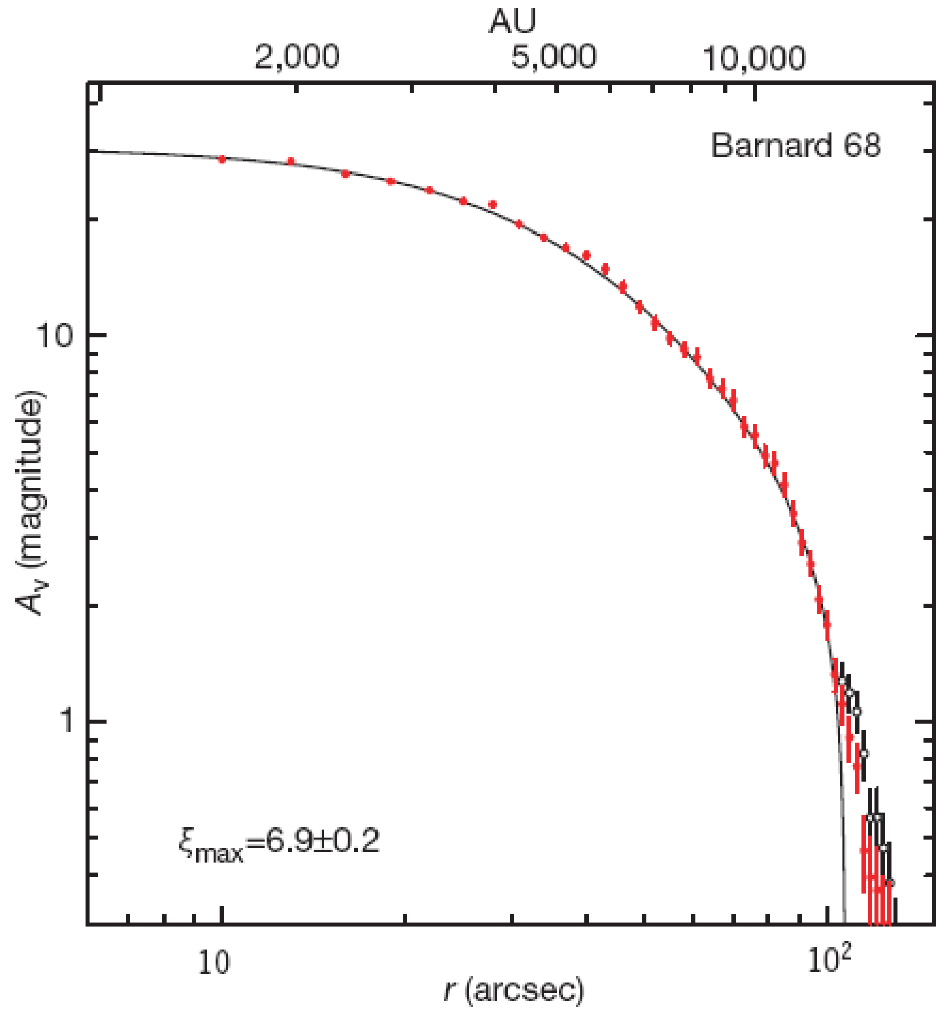
the solution of which is a Bonnor-Ebert sphere

Solution (numeric)



- density is flat at the core
- falls off with r^{-2} to the outside

B68 – a Bonnor-Ebert sphere



Mass

the mass of the BE sphere is

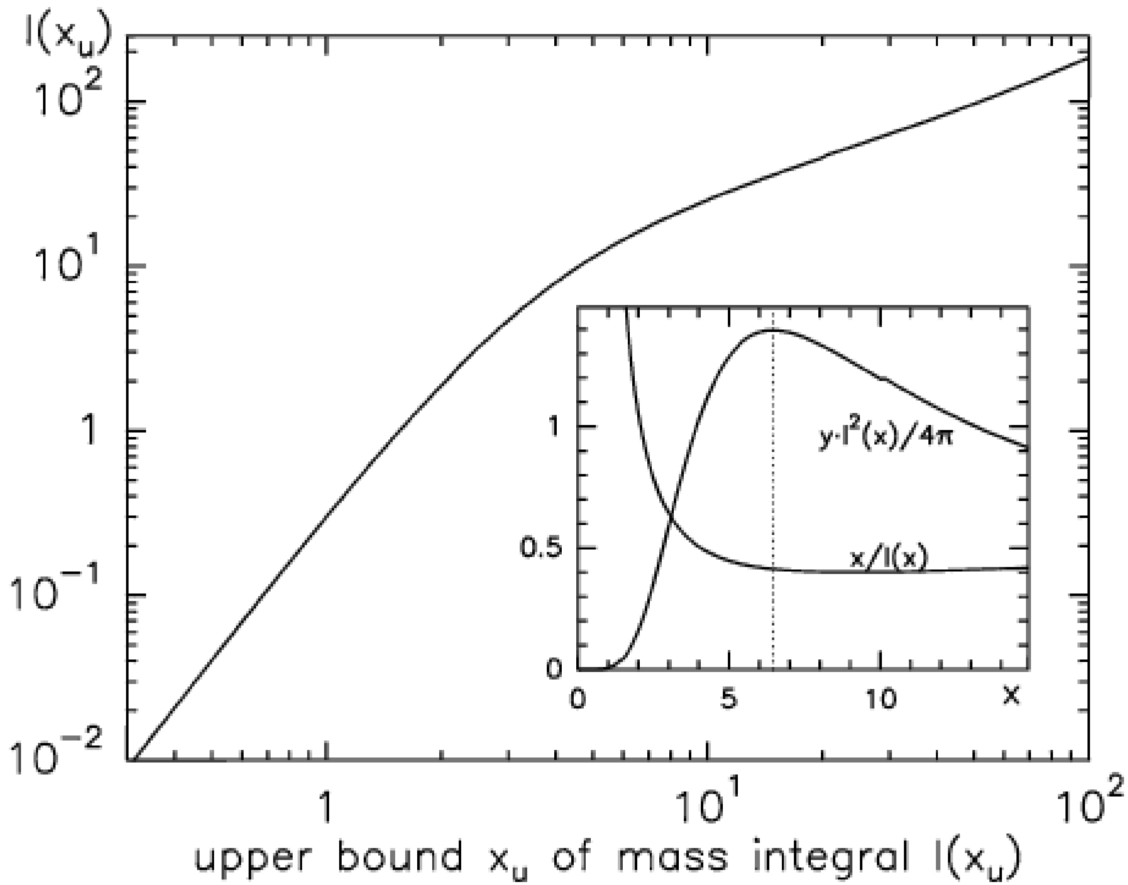
$$M = 4\pi \int_0^R r^2 \rho \, dr$$

$$M = \frac{1}{\sqrt{4\pi \rho_c}} \frac{c_s^3}{G^{3/2}} \int_0^{x_u} y \, x^2 \, dx$$

with $x_u = R\sqrt{4\pi G \rho_c / c_s^2}$

Relevant integral:

$$I(x_u) = \int_0^{x_u} y \, x^2 \, dx$$



Boundary

Problem: Solution only valid for infinite configuration

- At any outer edge, the mass would disperse from the thermal pressure.
- Cloud boundary must be consistent with external pressure.

at the outer edge, at $r = R$
the cloud is bound by the outer pressure P_{ext}
which is equal to the inner pressure at this point

$$P_{ext} = c_s^2 \rho(R)$$

Boundary

- The value of the outer pressure P_{ext} and radius "scale" the dimensionless BE-sphere to physical units.

Total radius and mass:

$$R = \frac{c_s^2}{\sqrt{2\pi G}} P_{ext}^{-1/2}$$

$$M(R) = \sqrt{\frac{2}{\pi}} \frac{c_s^4}{G^{3/2}} P_{ext}^{-1/2}$$

Stability

A cloud is stable if a pressure increase will be compensated by a size reduction: $\frac{\partial P_{ext}}{\partial R} < 0$

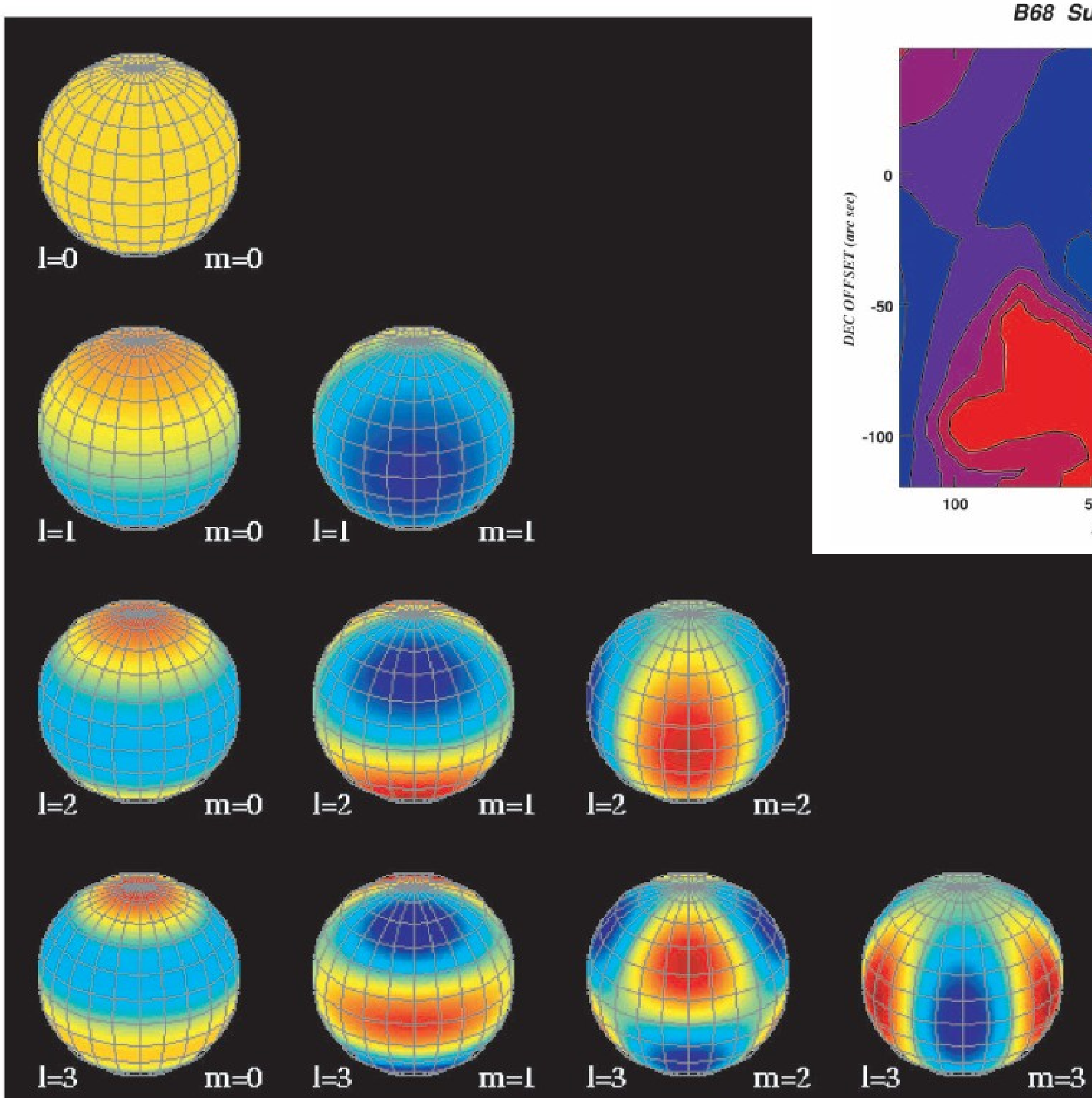
Detailed computations show:

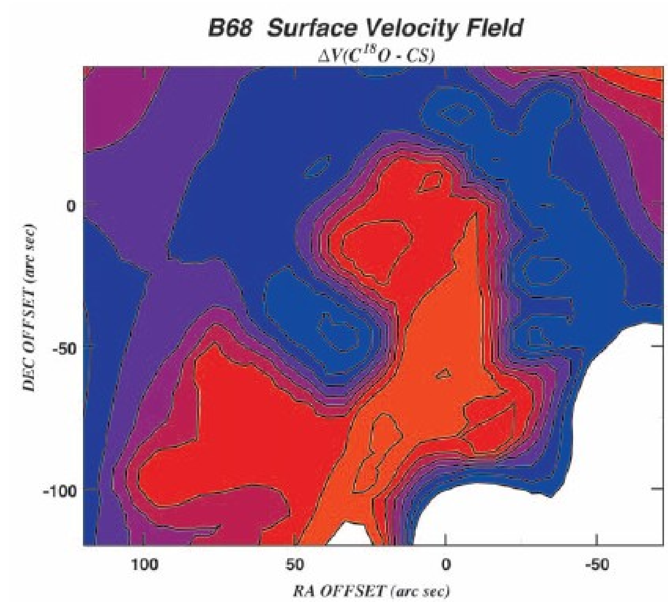
$$P_{crit} = 1.40 \frac{k^4}{G^3 m^4} \frac{T^4}{M^2}$$ maximum outer pressure

$$R_{crit} = 0.411 \frac{Gm}{kT} M$$ minimum radius for stability

when such a cloud is perturbed, it will oscillate around its equilibrium position

Oscillating B68





IG. 8.—False-color intensity maps of the real part of a series of low-order spherical harmonics. Visual inspection suggests that the $l = 2$, $m = 2$ mode :ars to correspond to the distribution of δV for B68, which is displayed in Fig. 7. (Figure courtesy of C. Pryke, University of Chicago.)

Critical mass

$$M_{crit} = 1.18 \frac{c_s^4}{G^{3/2}} P_{ext}^{-1/2}$$

FIG. 1.—Density distributions of bounded isothermal spheres. The outer radius of each sphere is given by the intercept of the corresponding curve with the abscissa. The curve marked "critical" denotes the sphere with the maximum mass consistent with hydrostatic equilibrium at a given external pressure. Hydrostatic spheres which are less centrally concentrated than the critical Bonnor-Ebert state are gravitationally stable; those which are more centrally concentrated are gravitationally unstable. In the limit of infinite central concentration, the latter spheres approach the singular solution.

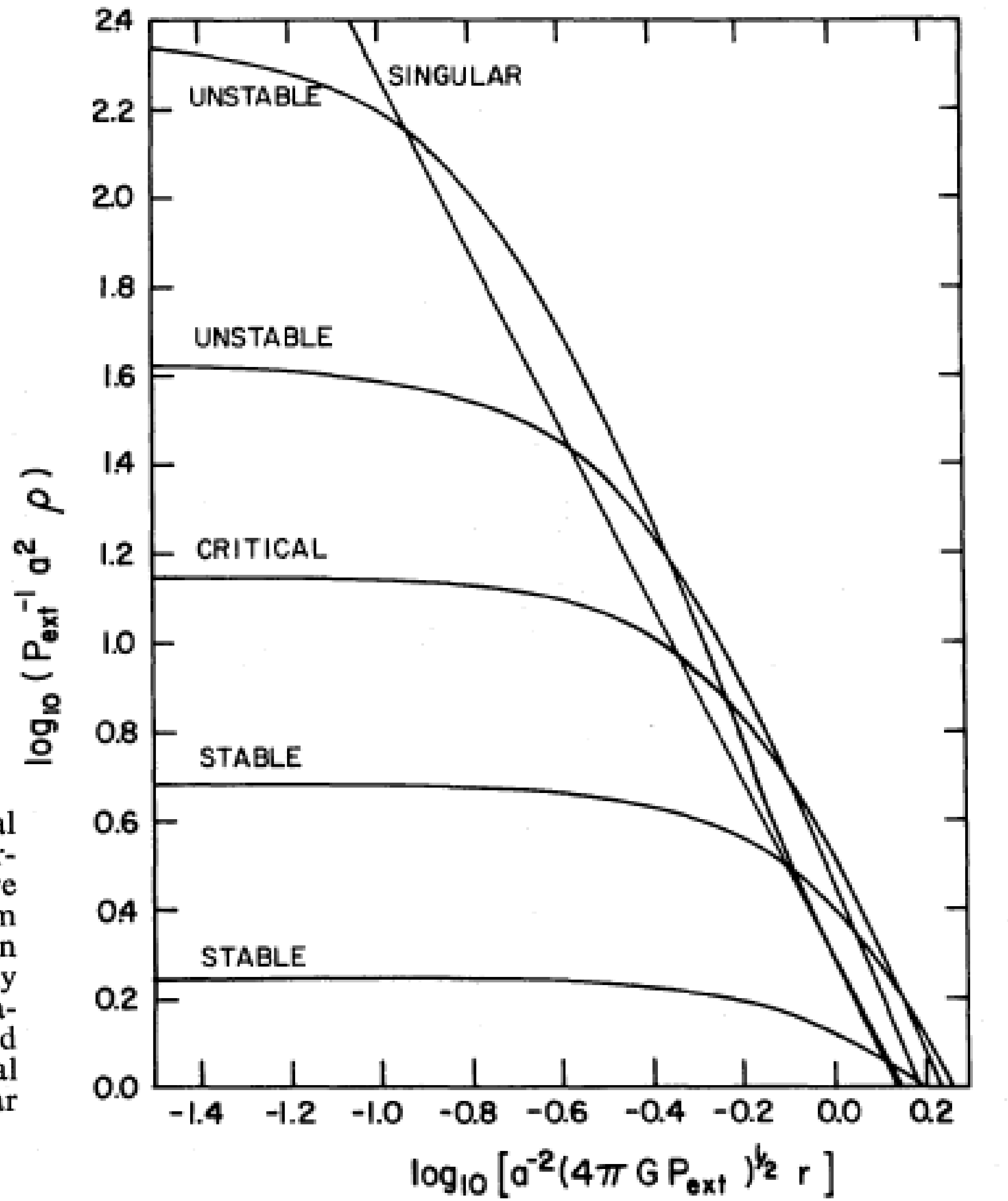
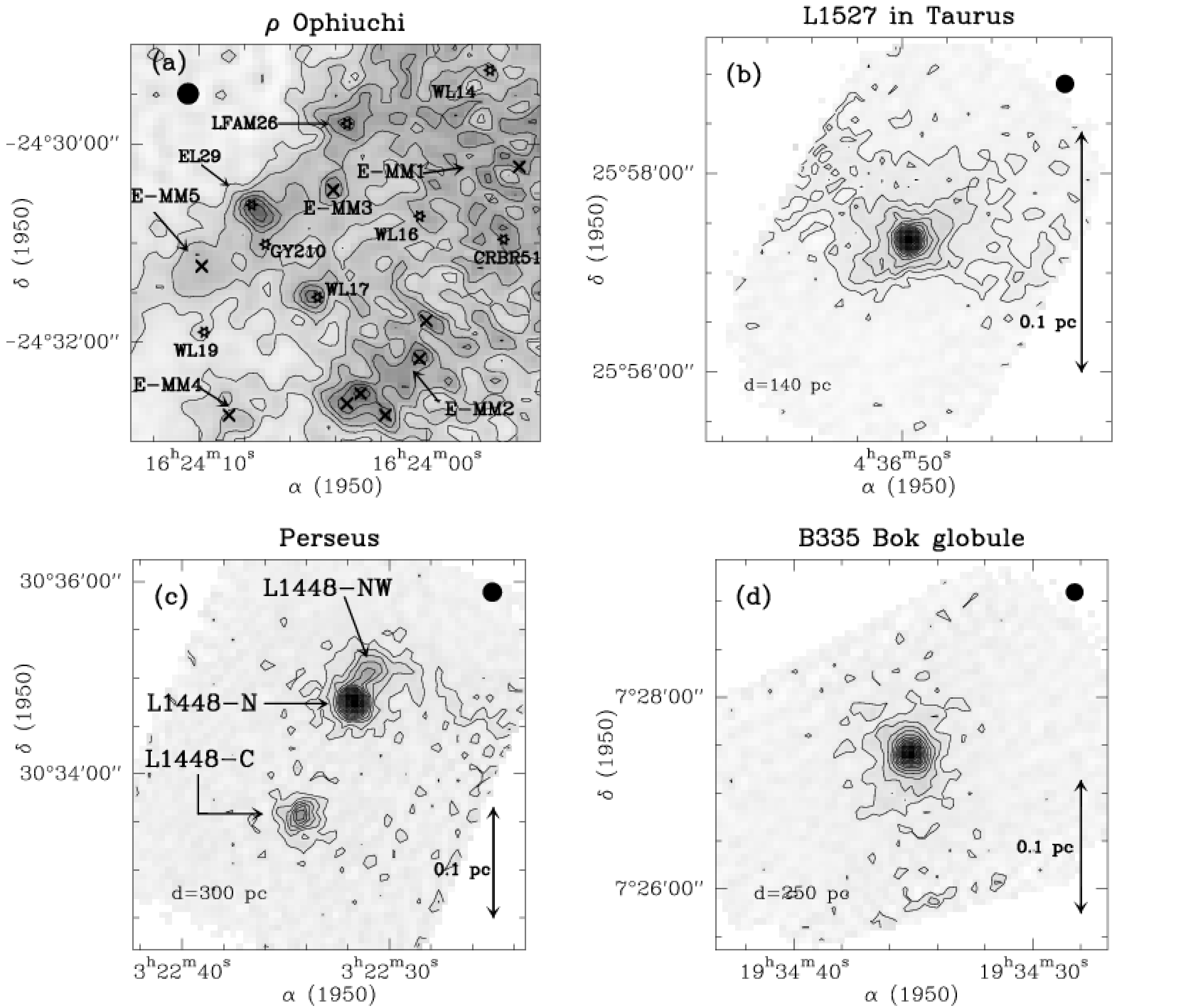


FIG. 1.—Density distributions of bounded isotherm

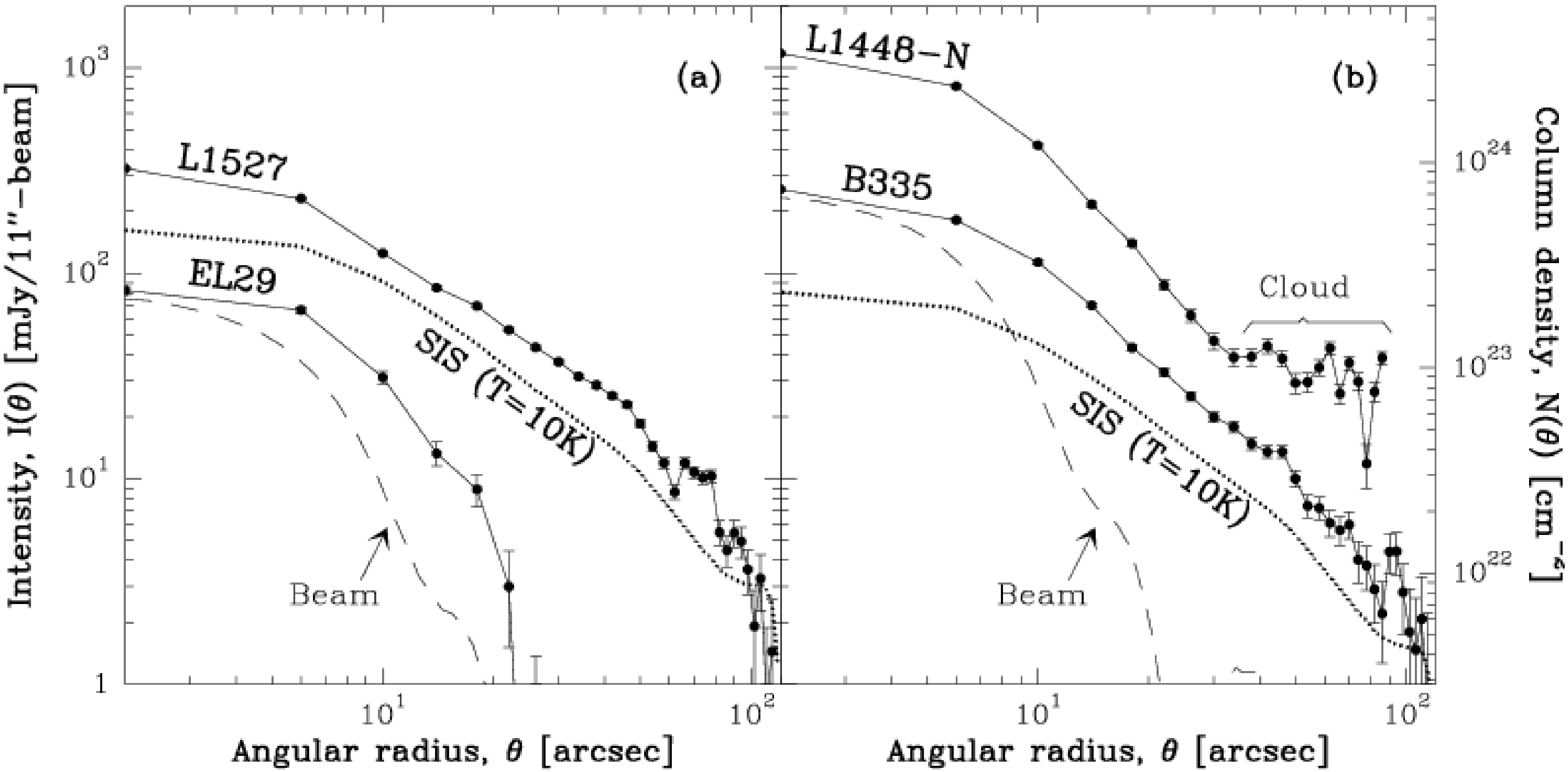
Remarks

- Pressures as in tenuous HII regions (nkT = 10^{-12} dyn cm^{-2}, that is a density of 1 cm^{-3} at a a temperature of 10,000 K) would allow a maximum mass of only 6 $M_{\odot}$
- That doesn't mean that more massive clouds can't be stable, it just means they need other ways of support (turbulent, magnetic, [rotation]) besides thermal
- unstable clouds will collapse and form stars

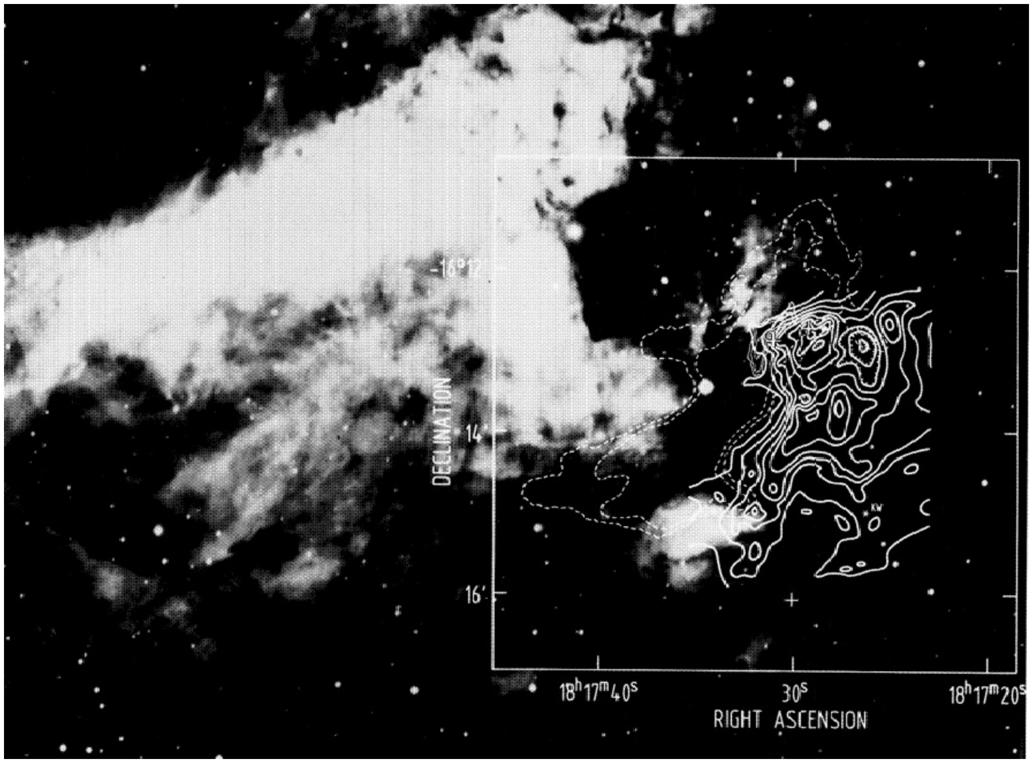
Structure of protostellar cores in ρ Oph: Motte & Andre (1999)



Structure of protostellar cores in ρ Oph: Motte & Andre (1999)



Clump size spectra



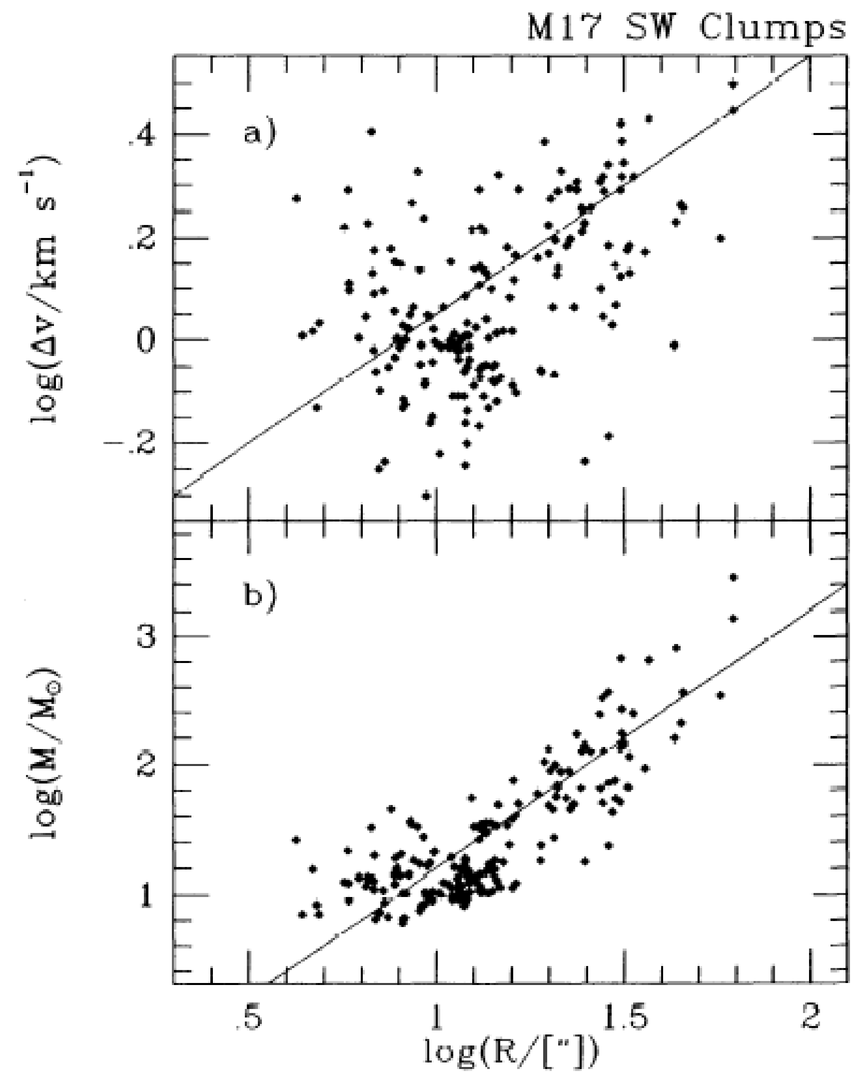


FIG. 13.—The distribution of clump masses vs. size and clump line widths vs. size for the M17 SW clumps. The straight lines correspond to the empirical laws $\Delta v \propto R^{0.5}$, and $M \propto R^2$.

- Wide distribution of clump masses
- Many cores close to virial equlibrium

(Stutzki & Güsten 1990)

Virial Theorem

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2T + 2U + 2\tau_{\text{int}} + M + \tau_m + W$$

with $I = \int_V \rho r^2 dV$ moment of inertia

and $T = \frac{1}{2}\int_V \rho v^2 dV$ kinetic (bulk) energy

and $U = \frac{3}{2}\int_V P\, dV$ internal (thermal) energy

and $\tau_{\text{int}} = \frac{1}{2}\oint_S x_i P \hat{n}_i dS$ surface pressure term

and $M = \frac{1}{8}\pi \int_V B^2 dV$ magnetic energy

and $\tau_M = \frac{1}{4}\pi \oint_S x_i B_i B_j \hat{n}_j dS$ magnetic stress at surface

and $W = -\int_V x_i \rho \frac{\partial \phi}{\partial x_i} dV$ gravitational energy

Virial equilibrium

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 0 \text{ virial equilibrium}$$

$$\text{and } \tau_{\text{int}} = \tau_M = 0$$

$$2T + 2U + M + W = 0$$

Thermal Support

$$\text{for } U = \frac{M\,R\,T}{\mu} \text{ (thermal energy)}$$

$$\frac{U}{|W|} = \frac{M\,R\,T}{\mu}\left(\frac{GM^2}{R}\right)^{-1}$$

$$= 3\times 10^{-3}\left(\frac{M}{10^5\,M_\odot}\right)^{-1}\left(\frac{R}{25\ pc}\right)\left(\frac{T}{15\ K}\right)$$

$$\frac{U}{|W|} \ll 1 \text{ for reasonable values: no thermal support}$$

Magnetic Support

$$\frac{M}{|W|} = \frac{|\boldsymbol{B}|^2 R^3}{6\pi} \left(\frac{GM^2}{R} \right)^{-1}$$

$$= 0.3 \left(\frac{B}{20\,\mu G} \right)^2 \left(\frac{R}{25\,pc} \right)^4 \left(\frac{M}{10^5 M_\odot} \right)^{-2}$$

$\frac{M}{|W|}$ close to 1: possible for support

Kinetic Energy

$$\frac{T}{|W|} = \frac{1}{2} M \Delta V^2 \left(\frac{GM^2}{R} \right)^{-1}$$

$$= 0.5 \left(\frac{\Delta V}{4\,\mathrm{km/s}} \right)^2 \left(\frac{R}{25\, pc} \right) \left(\frac{M}{10^5 M_\odot} \right)^{-1}$$

$$V_{vir} = \sqrt{\frac{G M}{R}} \text{ virial velocity dispersion}$$

many clouds observed with approximately virial velocity dispersions

Virial mass

- For cloud supported by turbulence one often assumes virial equilibrium and *U = M = 0*
- Then, one defines a virial mass

$$2T + W = 0$$

$$M_{vir} = \frac{\langle R \rangle \overline{v^2}}{G} \text{ virial Mass}$$

$\overline{v^2}$ is the 3-D rms

obtained from 1-D FWHM assuming isotropy

$$\frac{M}{M_\odot} = 250 \left(\frac{\Delta v_{FWHM}}{\text{km s}^{-1}} \right)^2 \left(\frac{R}{\text{pc}} \right)$$

Stability

- Often, one compares M_{vir} with some other mass (e.g. determined by dust measure-ments) and says
  - The cloud is stable, if $M_{vir} \geq M$
  - Unstable to collapse, if $M_{vir} \leq M$

Rests on a *lot* of assumptions

Jeans criterion

- James Jeans was the first to define a general criterion for stability
  - Infinitely extended uniform medium
  - Purely thermally supported
  - constant temperature, uniform density and initially at rest
    - unphysical, but useful order-of-magnitude estimate

Jeans criterion

- Consider infinite medium with constant density ρ_0:
  - Hydrodynamic equations for infinite homogeneous medium
  - Euler: $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} P - \vec{\nabla}\Phi$$
  - Continuity: $$\frac{\partial p}{\partial t} + \vec{v} \cdot \vec{\nabla}\rho + \rho\vec{\nabla} \cdot \vec{v} = 0$$
  - Poisson: $$\nabla^2\Phi = 4\pi G\rho$$

Jeans criterion

- Consider temporal evolution of a small pertubation ρ_1:
  - Induces pertubation in velocity field v_1, pressure p_1 and gravity field Φ_1

- Linear perturbation

$$\rho = \rho_0 + \rho_1, \quad P = P_0 + P_1 = v_S^2 \rho, \quad \Phi = \Phi_0 + \Phi_1, \quad \vec{v} = \vec{v}_0 + \vec{v}_1$$

- Resulting equations

$$\frac{\partial \vec{v}_1}{\partial t} = -\vec{\nabla}\left(\Phi_1 + v_S^2 \frac{\rho_1}{\rho_0}\right)$$

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \vec{\nabla} \cdot \vec{v}_1 = 0$$

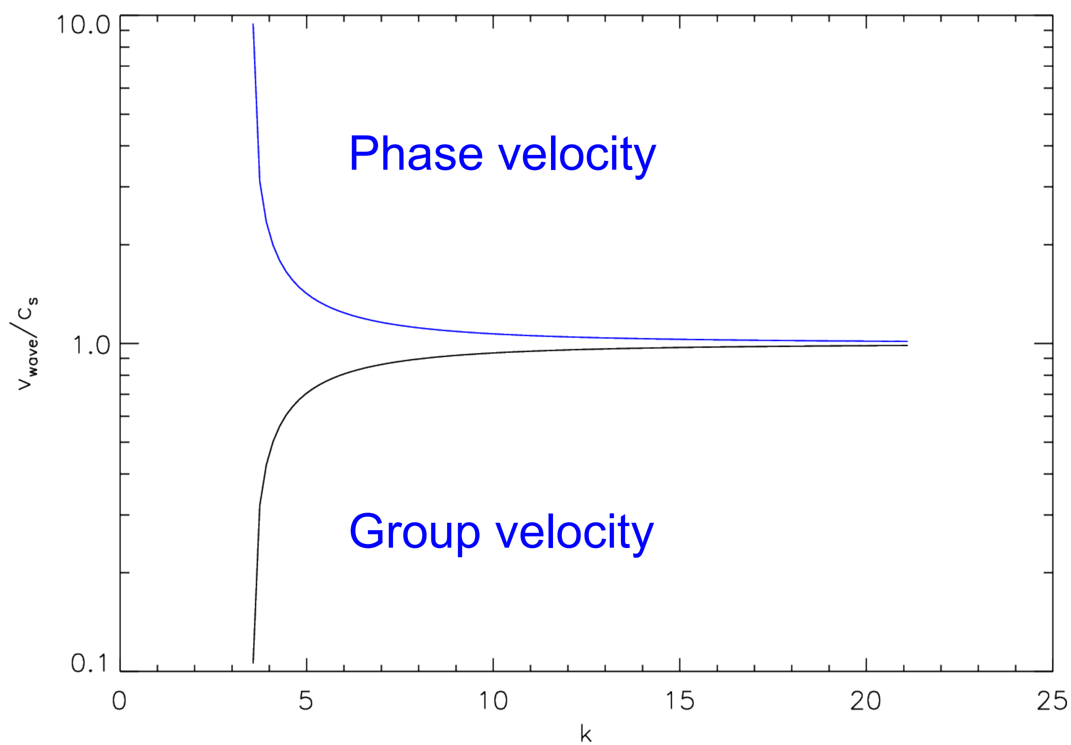
$$\nabla^2 \Phi_1 = 4\pi G \rho_1$$

Jeans criterion

- General solution of the form $\propto e^{i(kx+\omega t)}$

Resulting dispersion relation: $\omega^2 = k^2 v_S^2 - 4\pi G\rho_0$

- Oscillation for large *k*, exponential growth for small *k*
  - Jeans wavenumber $k_J^2 \equiv \frac{4\pi G\rho_0}{v_S^2}$
  - Jeans length $\lambda_J = \sqrt{\frac{\pi}{G\rho_0}}\, v_S$



- Real solution: density waves
- Imaginary solution: collapse or expansion
- Transition at k_J

Jeans criterion

The cloud is unstable if the pertubation grows exponentially, i.e. $\omega^2 < 0$ which leads to the critial wavenumber

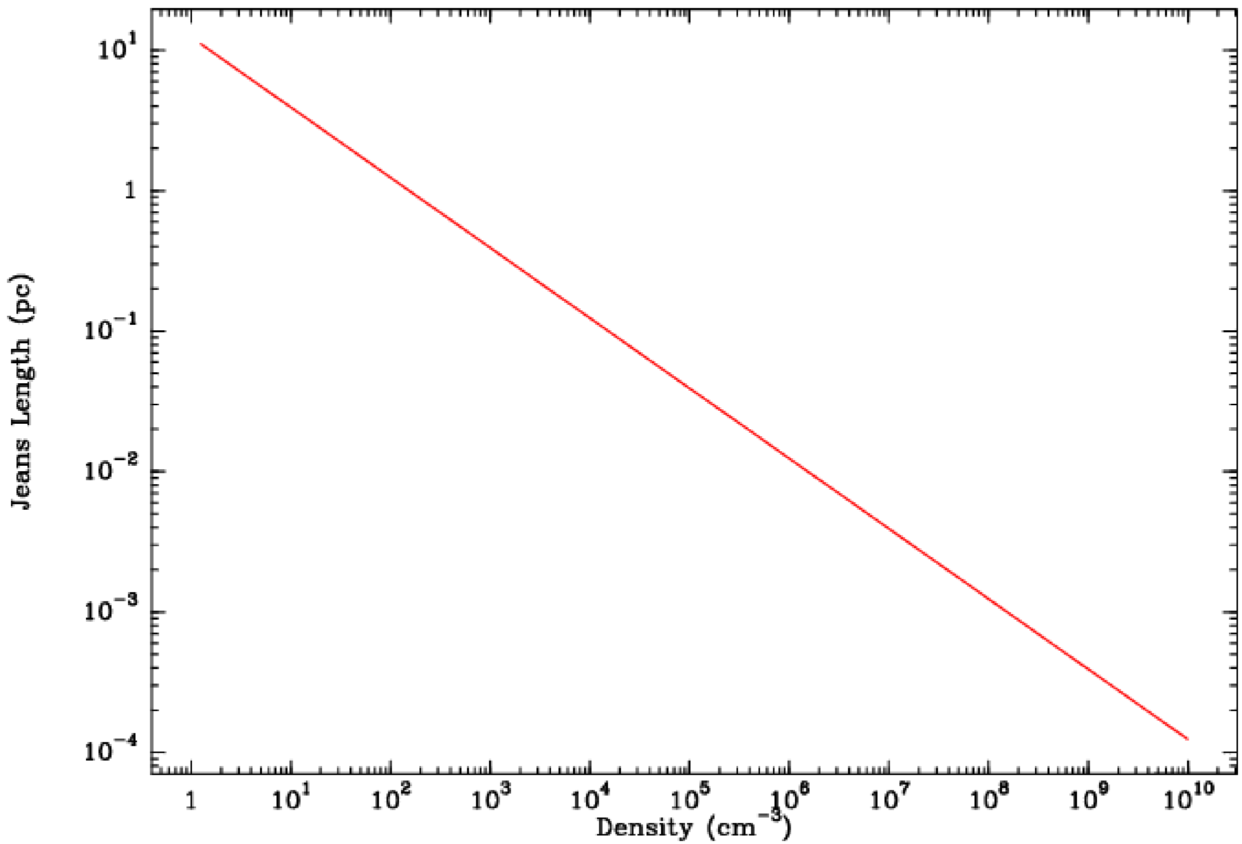
$$k_J = \frac{\pi}{L_J} = \sqrt{\frac{4\pi G m \rho_0}{kT}}$$

- Exact numerical factor depends on geometry. E.g., for a plane parallel slab $k_J^2 = \frac{2\pi G \rho_0(0)}{v_S^2}$

- A medium with a size > L_J allows instable pertubations to develop
- Clouds with L > L_J are prone to collapse.

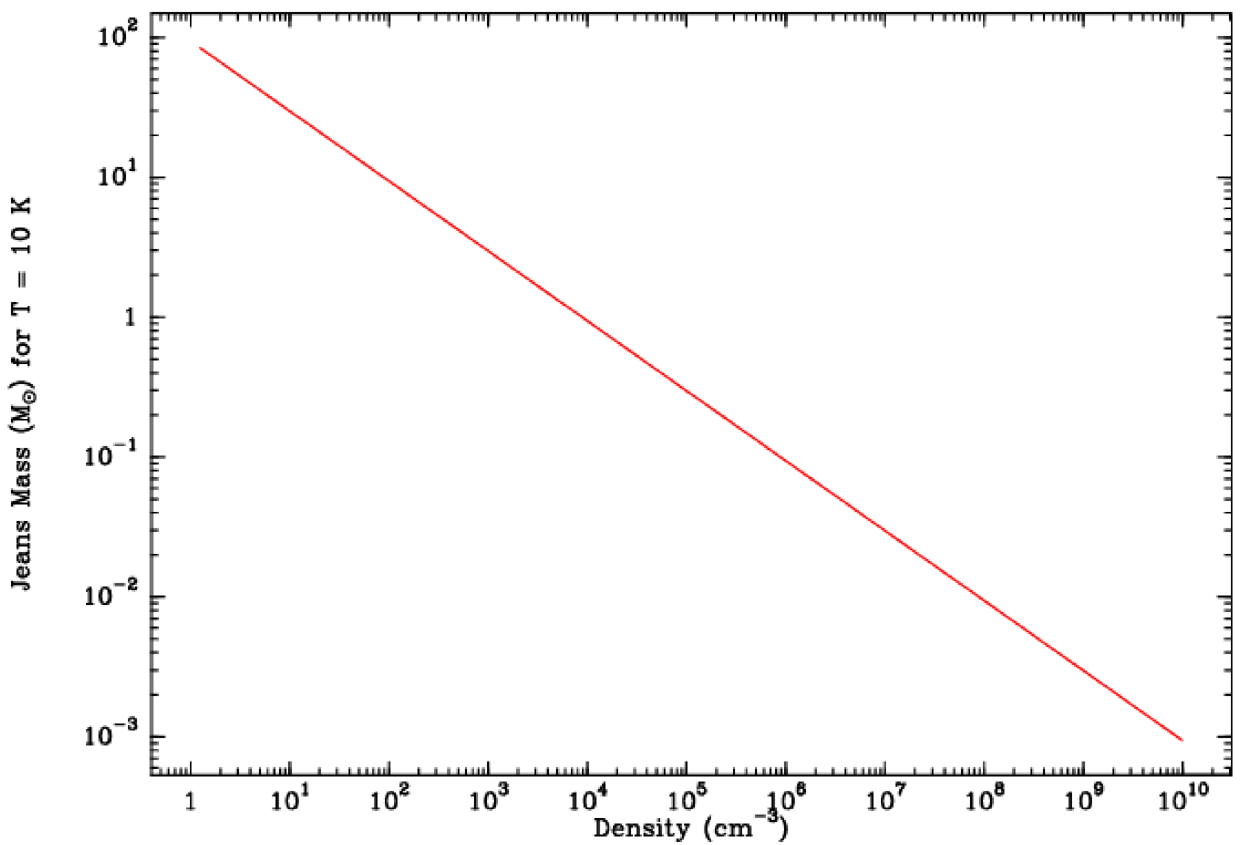
Jeans length

$$L_J = \sqrt{\frac{\pi k T}{4 G m \rho}} = 0.12 \sqrt{\frac{(T/10)[K]}{(n/10^4)[cm^{-3}]}} [pc]$$



Jeans Mass

$$M_J \approx L_J^3 \rho = \left(\frac{\pi k}{4 G m} \right)^{3/2} \sqrt{\frac{T^3}{\rho}} \approx \sqrt{\frac{(T/10\,K)^3}{(n/10^4\,cm^{-3})}} \, [M_\odot]$$



Jeans Mass

- Giant Molecular clouds: $T=20K$, $n=10^3\ cm^{-3}$
  $\rightarrow M_J = 10\ M_\odot$
  but: $M_{GMC} = 10^4\ M_\odot$ → all GMCs should collapse

- Should not be taken too seriously
  - other means of support
    - magnetic
    - turbulent – that can be included by using the turbulent velocity instead of the sound velocity

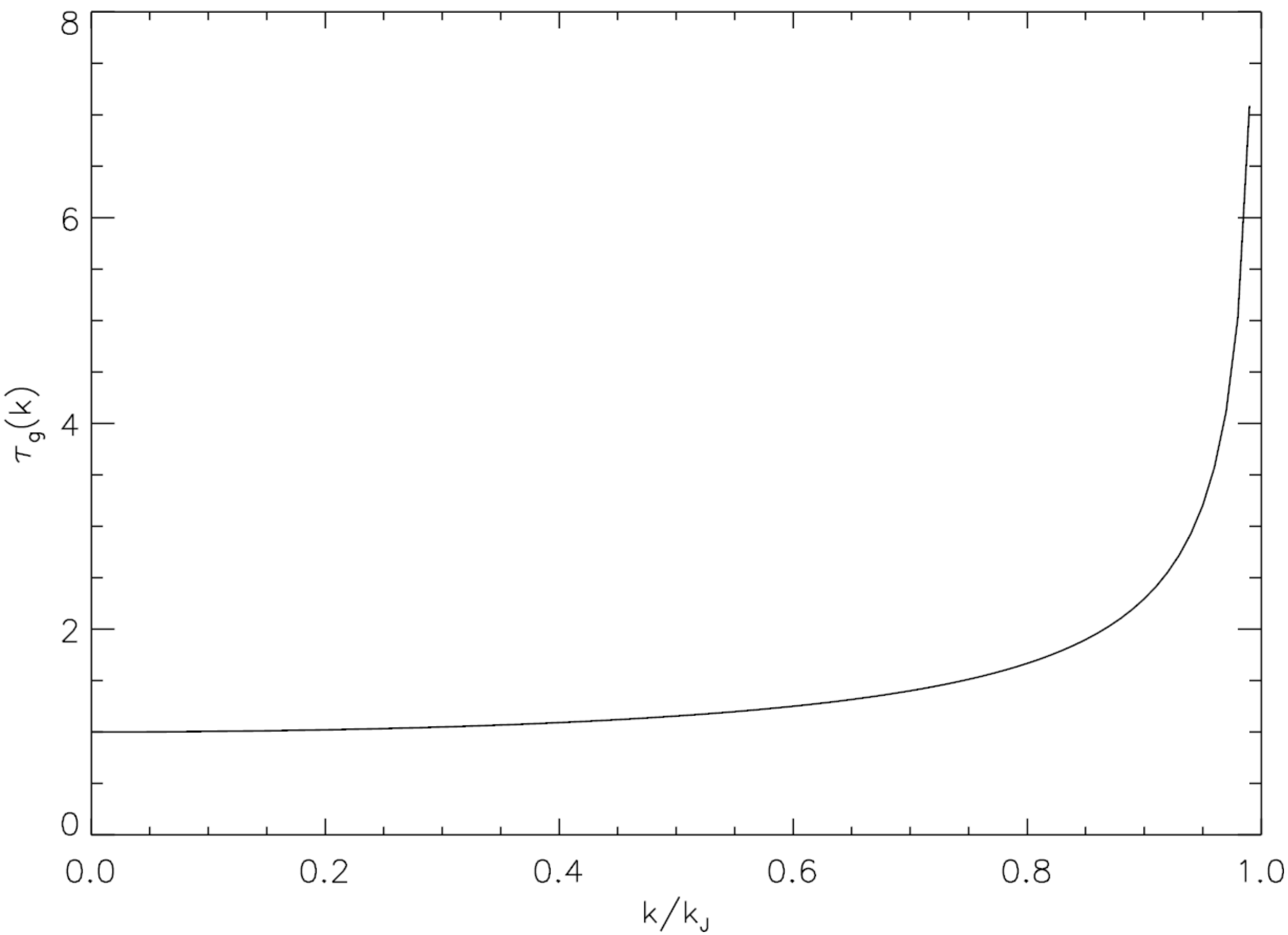
- Because the Jeans mass drops with densi-ty, a collapsing cloud will fragment

Free-fall time

- Compute time scale from Jeans dispersion relation:

$$\tau_g = \left[4\pi G\rho_0 - c_s^2|\vec{k}|^2\right]^{-1/2}$$

- Fastest growth for k → 0
- $\tau_{ff} = \sqrt{\frac{1}{4\pi G\rho}}$
- Only density dependent
- Every mass > M_J must collapse in τ_{ff}



Free-fall time

- Alternative computation
  - Assuming finite cloud (virial approach)

$$\frac{1}{2}\frac{d^2 I}{dt^2} = W \quad \text{free gravitational collapse}$$

$$\frac{1}{2}\frac{d^2 I}{dt^2} = -\frac{G M^2}{R} \quad (R \text{ is a characteristic Radius})$$

$$\text{and } I = MR^2$$

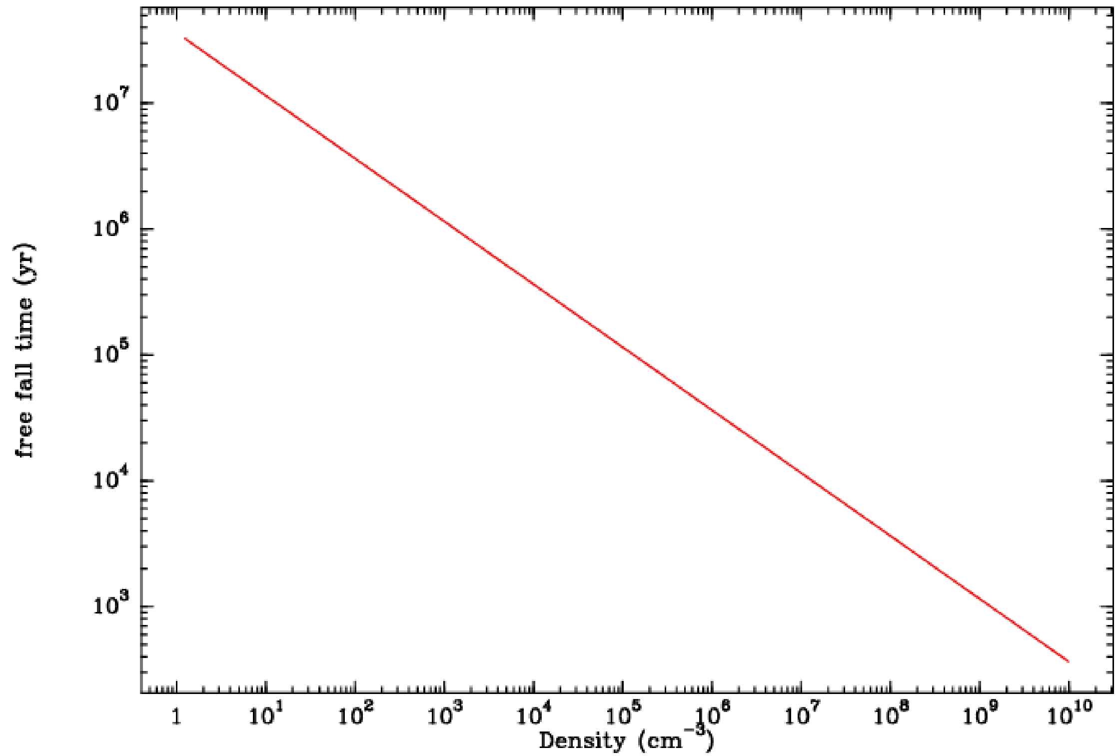
$$t_{ff} = \sqrt{\frac{R^3}{GM}} = 7\times 10^6\,\text{yr}\left(\frac{M}{10^5 M_\odot}\right)^{-1/2}\left(\frac{R}{25\,pc}\right)^{3/2}$$

free-fall time

- Same result – fast collapse $\propto 1/\sqrt{\rho}$

Free-fall time

Depends only on density, not on total mass or size



- Free fall time

$$t_{ff} = \left(\frac{3\pi}{32}\frac{1}{G\rho}\right)^{1/2} = 1.4 \times 10^5 \left(\frac{2n(\mathrm{H}_2)}{10^5\,\mathrm{cm}^{-3}}\right)^{-1/2} \mathrm{yr} \approx 3 \times 10^6\,\mathrm{yr}$$

- Milky Way cloud mass ~4×10^9 M_{Sun}
- Theoretical star formation rate M_{cloud}/t_{ff} ~ 1400 M_{Sun}/yr
- Observed star formation rate across the MW ~ 3 M_{Sun}/yr
  - cf. star burst galaxy: tens to hundreds M_{Sun}/yr

Star Formation and Molecular Clouds

- Star Formation Efficiency in Clouds
  - 1% to 4% of mass with A_V>2 in dense cores
  - 2% to 4% in stars
  - Cloud depletion time 40-100 Myrs

- Star Formation Efficiency in Cores
  - About 25% in dense gas
  - Core depletion time 0.5 to 3 Myr

What does it mean?

1) Star formation could be slow, but efficient
   - At any given time, only a few % of the mass are made into stars
   - But eventually, most of the cloud gets trans-formed into stars
2) Star formation could be fast, but inefficient
   - Molecular clouds form rapidly
   - A few % of the mass is transformed into stars
   - The rest is dispersed again

- Relationship with Molecular Cloud lifetimes

Model 1: slow star formation

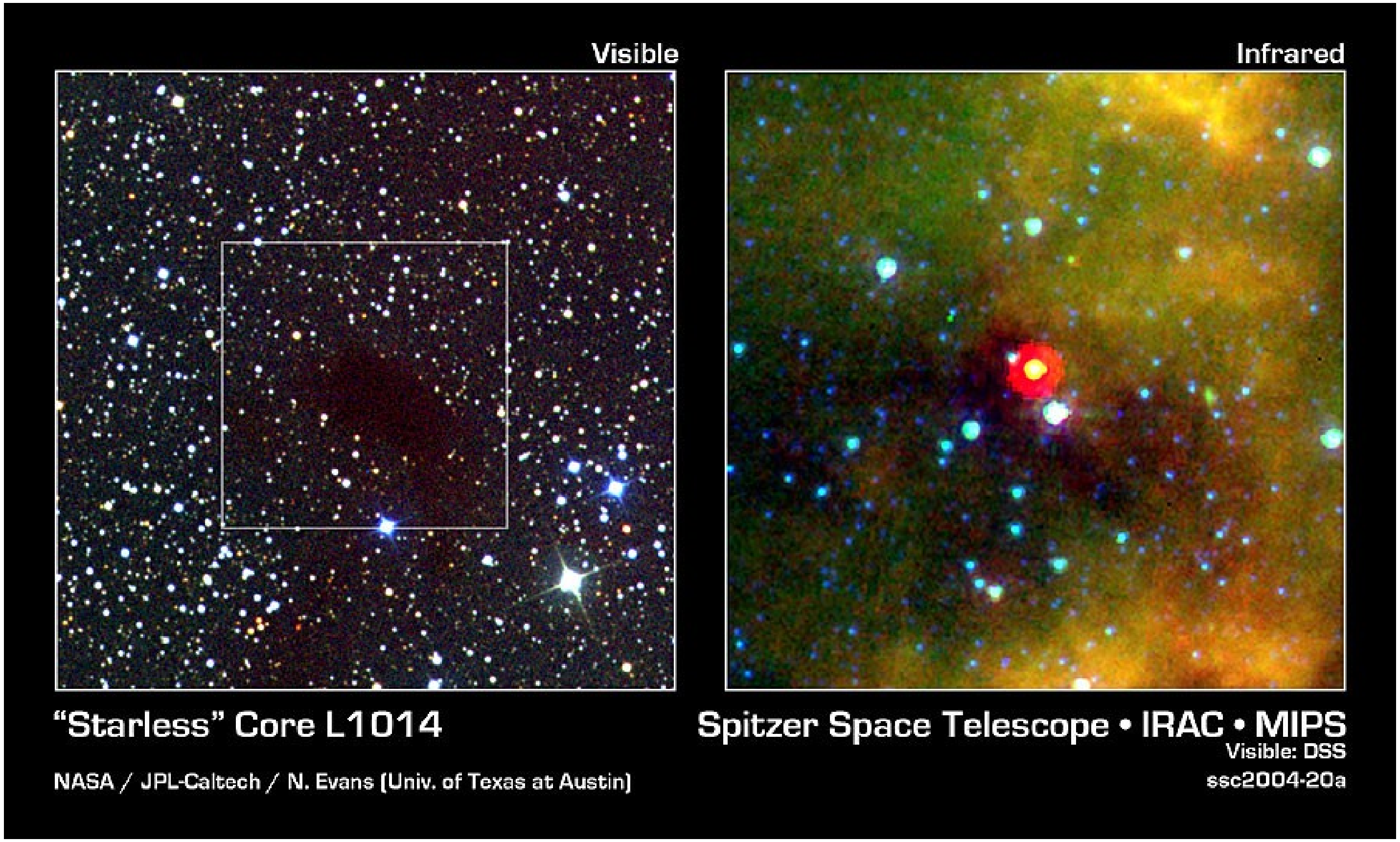
- Molecular clouds are long lived (few 10^7 yrs)
- ISM is in an equilibrium state
- Magnetic fields prevent rapid collapse of cores
- Stay in quasi-static equilibrium for a long time
- Collapse possible through ambipolar diffusion
- Filamentary appearance: magnetic fields

Model 2: fast star formation

- Molecular clouds short lived (few 10^6 yrs at most)
- ISM is not in any equilibrium, particularly not virial
- Rapid turbulent fragmentation: solves the problem of support for turbulence
- Star Formation is very dynamic
- Some cores collapse and form stars
- Some cores disperse again
- Filamentary appearance: shocks

Arguments for rapid star formation

- Few starless cores observed



- No magnetically subcritical cores observed
- Small age spread in stellar associations
- Young chemistry supports young clouds

Global scenario

- Model 1: Retarding effects
  - turbulent pressure
  - Rotation → angular momentum transfer
  - magnetic fields are significant

- Model 2: No static cores exist!
  - Large scale turbulent flows
    → external terms τ_{int}, τ_M exceed internal terms
  - Continuous "stirring" of turbulence

Models: colliding flows

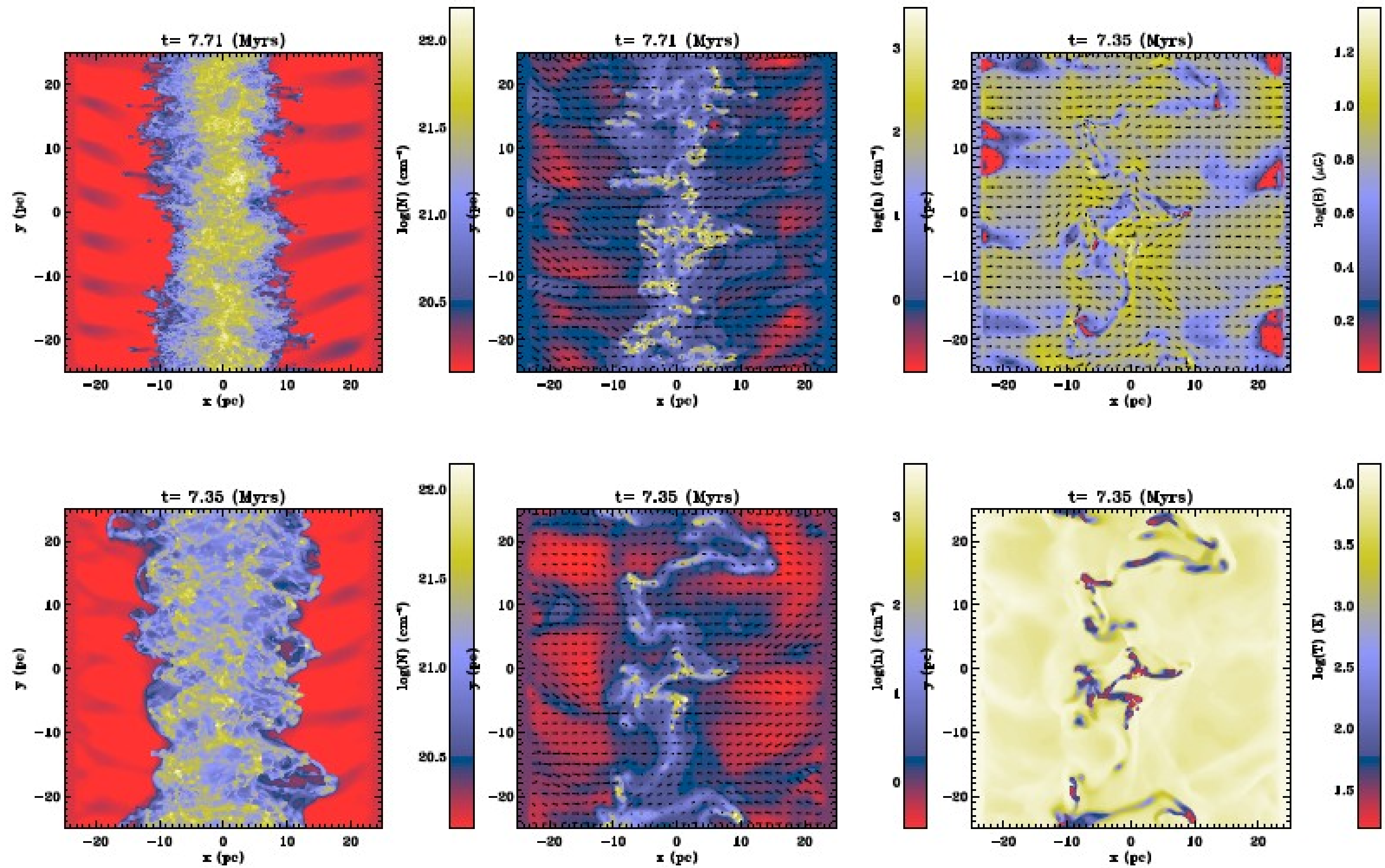
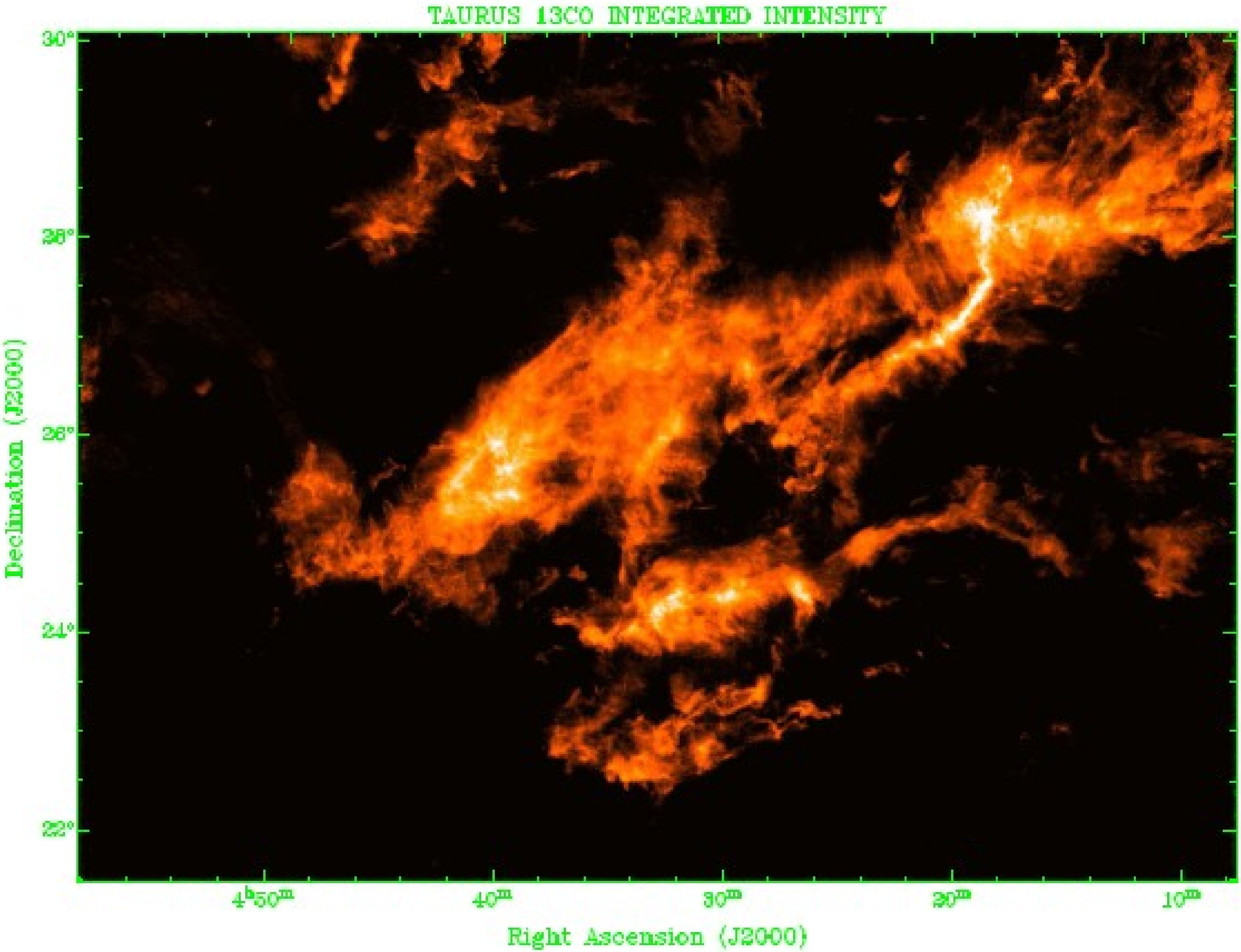


Fig. 2. *Top* (*bottom*) *panel of left column* shows the column density of the hydro (MHD) case. *Top* (*bottom*) *panel of middle column* shows the density and velocity fields in the $z = 0$ plane of the hydro (MHD) case. *Top panel of right column* shows the magnetic intensity and its xy-components (indicated as arrows) in the $z = 0$ plane. *Bottom panel of right column* shows the temperature in the $z = 0$ plane of the MHD case.

Hennebelle et al. 2008


TAURUS 13CO INTEGRATED INTENSITY
30°
28°
26°
24°
22°
Declination (J2000)
4h50m
40m
30m
20m
10m
Right Ascension (J2000)

Dynamics of collapse

- Larson-Penston solution
- Inside-Out collapse: never static

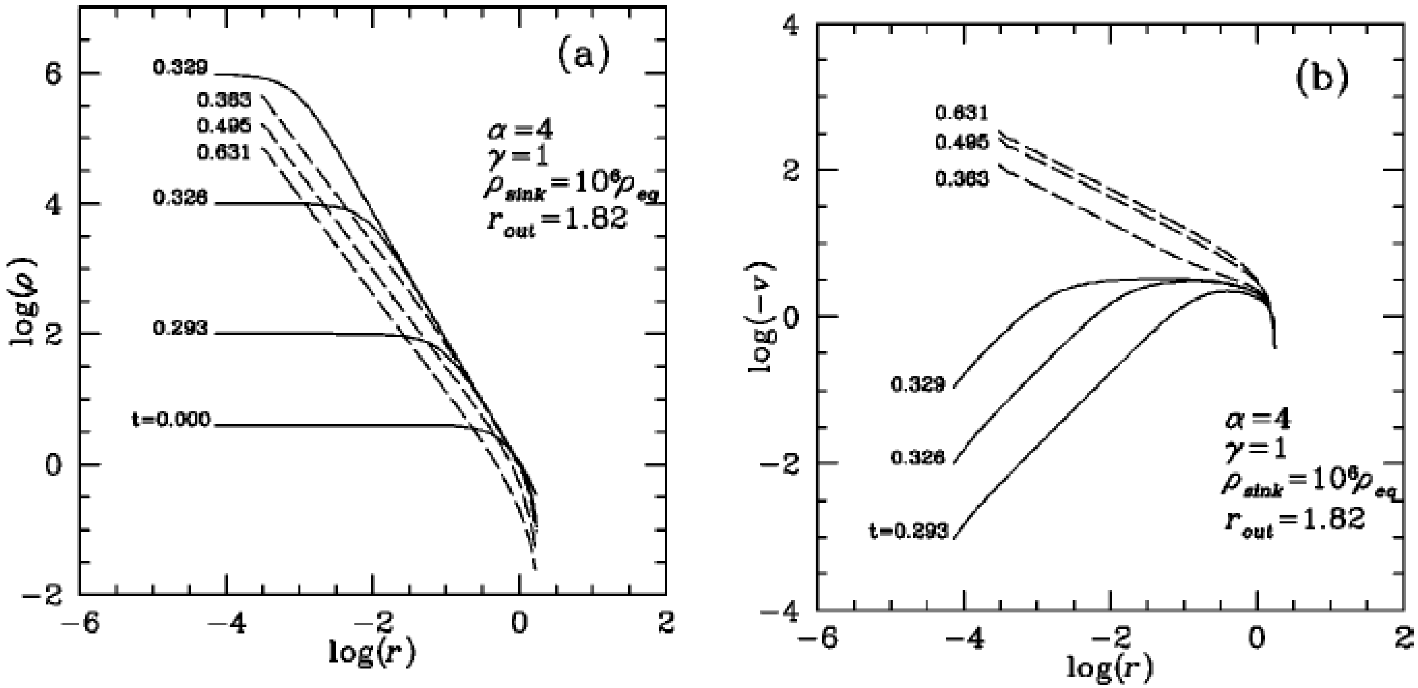


FIG. 7. Radial density profile (a) and infall velocity profile (b) at various stages of dynamical collapse. All quantities are given in normalized units. The initial configuration at $t=0$ corresponds to a critical isothermal ($\gamma=1$) Bonnor-Ebert sphere with outer radius $r_{out}=1.82$. It has $\alpha=4$ times more mass than allowed by hydrostatic equilibrium and therefore begins to contract. The numbers on the left denote the evolutionary time and illustrate the "runaway" nature of collapse. Since the relevant collapse time scale, the free-fall time τ_{ff}, scales with density as $\tau_{ff} \propto \rho^{-1/2}$ central collapse speeds up as ρ increases. When density contrast reaches a value of 10^6 a "sink" cell is created in the center, which subsequently accretes all incoming matter. This time roughly corresponds to the formation of the central protostar and allows for following its subsequent accretion behavior. The profiles before the formation of the central point mass are indicated by solid lines, and for later times by dashed lines. From Ogino *et al.*, 1999.

Dynamics of collapse

TABLE II. Properties of the Larson-Penston solution of isothermal collapse.

	Before core formation ($t<0$)	After core formation ($t>0$)
Density profile	$\rho \propto (r^2+r_0^2)^{-1}$ ($r_0 \to 0$ as $t \to 0_-$) flattened isothermal sphere	$\rho \propto r^{-3/2}$, $r \to 0$ $\rho \propto r^{-2}$, $r \to \infty$
Velocity profile	$v \propto r/t$ as $t \to 0_-$ $v \approx -3.3c_s$, $r \to \infty$	$v \propto r^{-1/2}$, $r \to 0$ $v \approx -3.3c_s$, $r \to \infty$
Accretion rate		$\dot{M} = 47c_s^3/G$

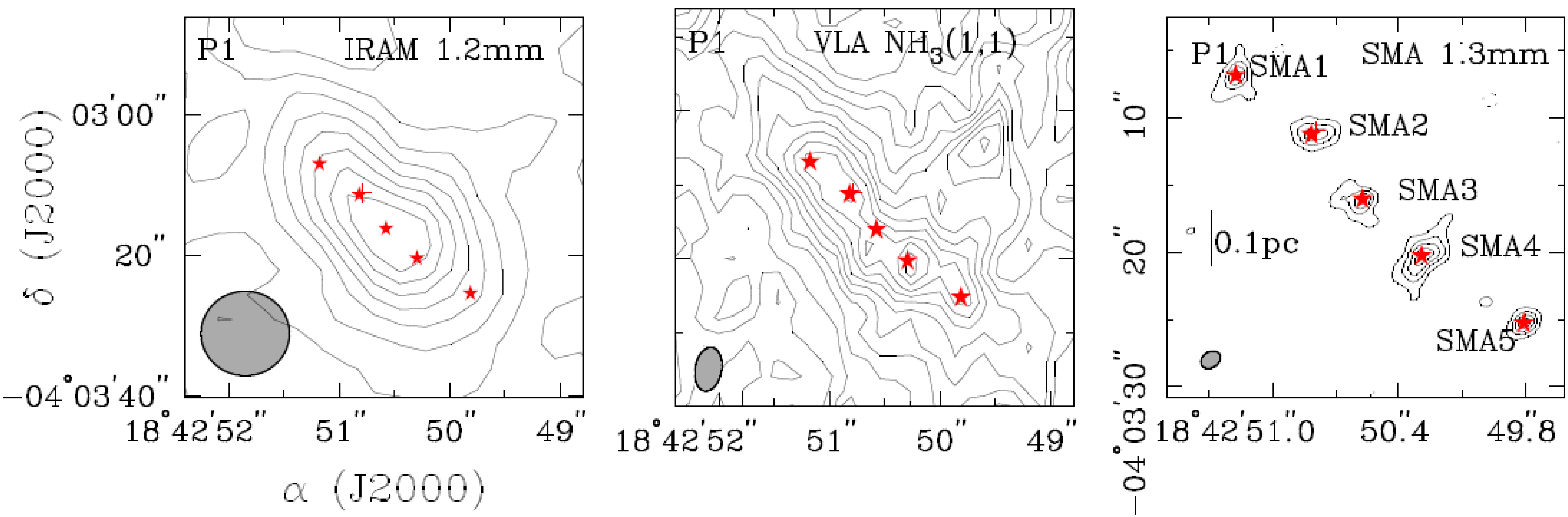
- Run-away collapse
- Higher density → faster collapse → higher density

Turbulent fragmentation

- High density regions always evolve fastest
- The density increase further reduces the Jeans mass
- Subunits become gravitationally instable and collapse independently
- Break-up of collapse into smaller and smaller entities → fragmentation
- End of cascade determined by
  - Dissipation scale from molecular viscosity
  - Ambipolar diffusion scale

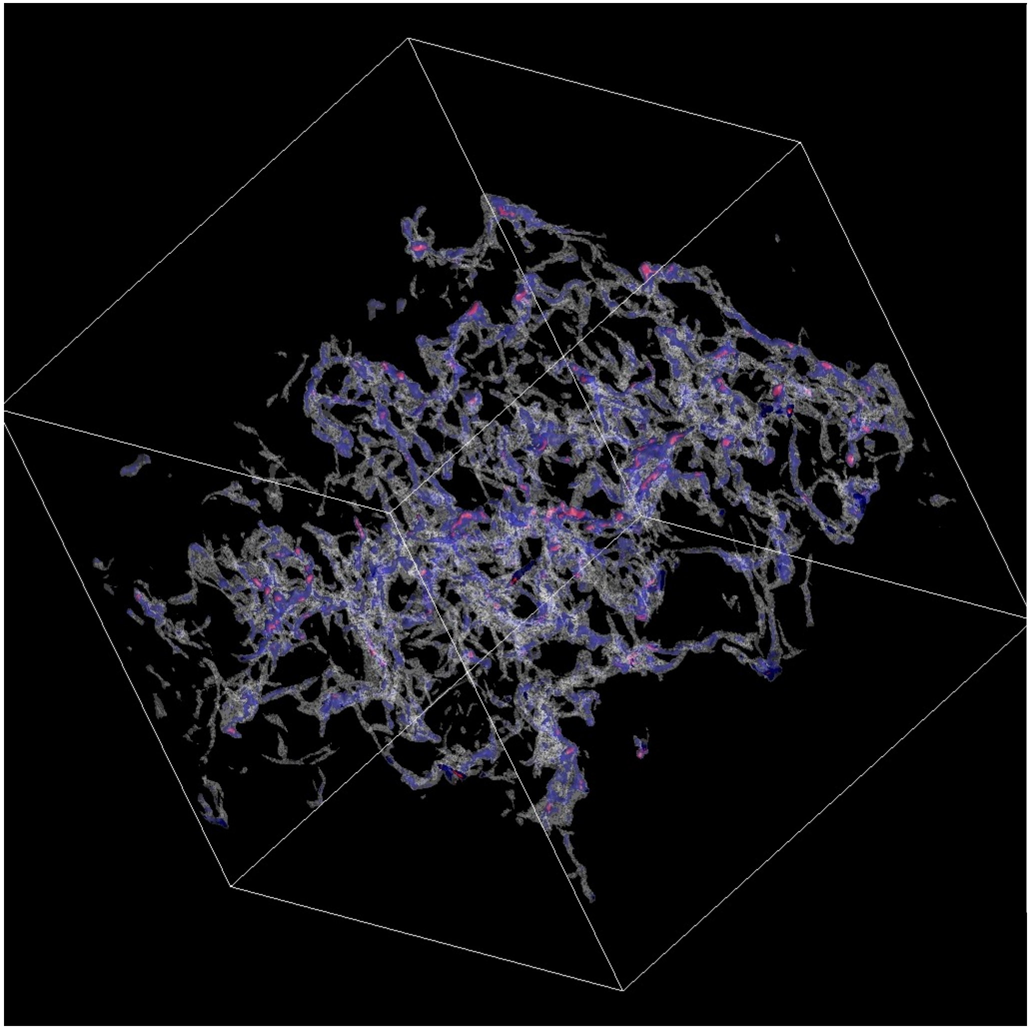
Observational evidence for Fragmentation

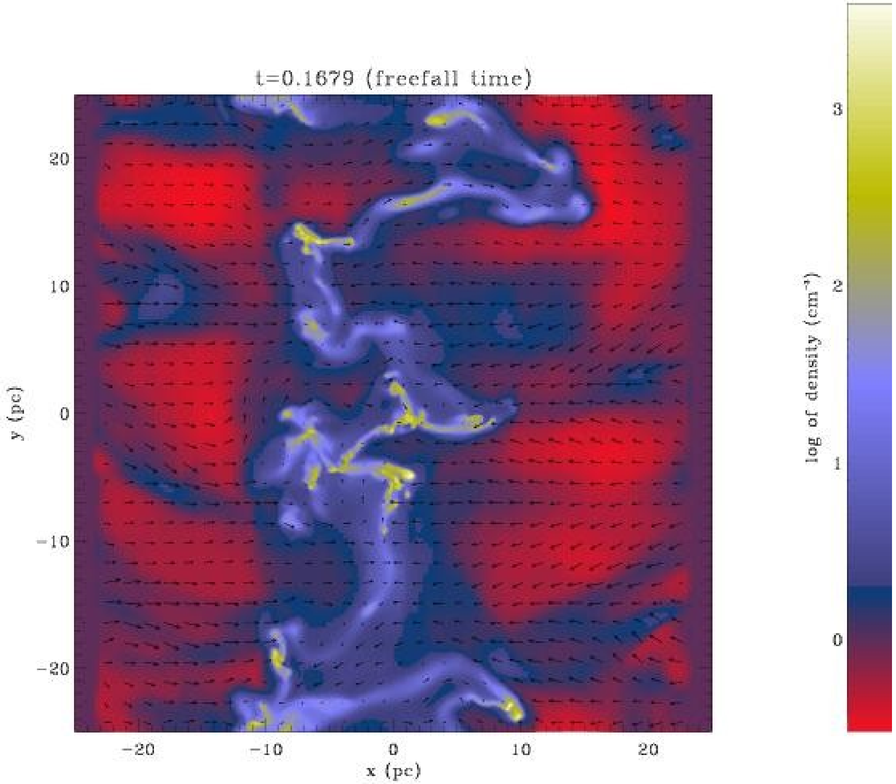
$M \approx 10\ M_{Jeans}$



Zhang et al. 2009

Turbulent cloud collapse





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