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Effective-medium theories for cosmic dust grains

Erratum

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Abstract. The treatment of the effective medium theories with magnetic dipole term in Sect. 2.2 of the original paper (Ossenkopf V. 1991, A&A 251, 210) is inconsistent since the direct influence of the permeability on the refractive indices was neglected. The equations in Sect. 2.2 and the results of their application in Sect. 3.1 have to be revised.

Although the magnetic dipole effects in an inhomogeneous medium were taken into account in the scattering matrix method by an effective permeability μ^*, the direct influence of this permeability on the refractive indices of the effective medium was not considered. In a consistent treatment we have to substitute ϵ in all equations of Sect. 2.2 by $\epsilon \times \mu$ as the correct square of the refractive indices for all components and the effective medium. Also the size parameter x has to be corrected by a factor $\sqrt{\mu^*}$. Then, Eq. (16) becomes completely symmetric in the electric and magnetic contributions and we obtain for the first terms of the scattering amplitude

$$\begin{aligned}
a_1 &= \frac{2\mathrm{i}}{3}\,\frac{\epsilon^\sigma-\epsilon^*}{\epsilon^\sigma+2\epsilon^*}x^3+\frac{\mathrm{i}}{5}\,\frac{\mu^*\epsilon^{\sigma\,2}+\mu^\sigma\epsilon^{\sigma\,2}-6\mu^*\epsilon^*\epsilon^\sigma+4\mu^*\epsilon^{*\,2}}{\mu^*\,(\epsilon^\sigma+2\epsilon^*)^2}x^5\\
b_1 &= \frac{2\mathrm{i}}{3}\,\frac{\mu^\sigma-\mu^*}{\mu^\sigma+2\mu^*}x^3+\frac{\mathrm{i}}{5}\,\frac{\epsilon^*\mu^{\sigma\,2}+\epsilon^\sigma\mu^{\sigma\,2}-6\epsilon^*\mu^*\mu^\sigma+4\epsilon^*\mu^{*\,2}}{\epsilon^*\,(\mu^\sigma+2\mu^*)^2}x^5\\
a_2 &= \frac{\mathrm{i}}{15}\,\frac{\epsilon^\sigma-\epsilon^*}{2\epsilon^\sigma+3\epsilon^*}x^5\\
b_2 &= \frac{\mathrm{i}}{15}\,\frac{\mu^\sigma-\mu^*}{2\mu^\sigma+3\mu^*}x^5,
\end{aligned}\tag{16}$$

where ϵ^σ and ϵ^* denote the dielectric functions and μ^σ and μ^* the relative permeabilities of the subgrain σ and the environment, respectively.

In the leading x^3-terms the dielectric function and the magnetic permeability are completely decoupled. For particles small compared to the wavelength ($x\ll 1$), where we only have to consider the first term in a_1 and the first two terms in b_1, the condition from Eq. (15) that $\sum_\sigma a_1^\sigma = \sum_\sigma b_1^\sigma = 0$ gives an equation for ϵ^* that does not depend on the magnetic properties of the grains. Therefore, the effective dielectric function will not be influenced by any quantities from b_1. However, as the effective permeability is mainly determined by the second b_1-term, it also depends on the dielectric functions of the materials. The induced magnetic dipole term is translated into an effective permeability μ^* different from 1 even if all the constituents are nonmagnetic materials.

The corrected expressions for Eq. (17) describing the contributions for core-mantle grains are

$$\begin{aligned}
a_1 &= \frac{2\mathrm{i}}{3}\,\frac{\Sigma_\epsilon(\epsilon^{\mathrm{m}}-\epsilon^*)+r\Delta_\epsilon(\epsilon^*+2\epsilon^{\mathrm{m}})}{\Sigma_\epsilon(\epsilon^{\mathrm{m}}+2\epsilon^*)+2r\Delta_\epsilon(\epsilon^{\mathrm{m}}-\epsilon^*)}x^3\\
b_1 &= \frac{2\mathrm{i}}{3}\,\frac{\Sigma_\mu(\mu^{\mathrm{m}}-\mu^*)+r\Delta_\mu(\mu^*+2\mu^{\mathrm{m}})}{\Sigma_\mu(\mu^{\mathrm{m}}+2\mu^*)+2r\Delta_\mu(\mu^{\mathrm{m}}-\mu^*)}x^3\\
&\quad+\mathrm{i}/5\big(1-\mu^{\mathrm{m}}/(4\epsilon^*)\times\big[5(\Sigma_\mu-4r\Delta_\mu)^2(\mu^{\mathrm{m}}\epsilon^{\mathrm{m}}-\mu^*\epsilon^*)\\
&\qquad-9{\Sigma_\mu}^2(\mu^{\mathrm{m}}\epsilon^{\mathrm{m}}-5\mu^*\epsilon^*)\\
&\qquad-36r^{5/3}\mu^{\mathrm{m}}[\mu^{\mathrm{c}2}(\epsilon^{\mathrm{c}}+\epsilon^{\mathrm{m}})+(4\mu^{\mathrm{m}2}-6\mu^{\mathrm{c}}\mu^{\mathrm{m}})\epsilon^{\mathrm{m}}]\big]\\
&\qquad\times[\Sigma_\mu(\mu^{\mathrm{m}}+2\mu^*)+2r\Delta_\mu(\mu^{\mathrm{m}}-\mu^*)]^{-2}\big)x^5\\
&\quad\text{with } \Sigma_\epsilon=\epsilon^{\mathrm{c}}+2\epsilon^{\mathrm{m}}\,,\ \Delta_\epsilon=\epsilon^{\mathrm{c}}-\epsilon^{\mathrm{m}}\\
&\quad\text{and } \Sigma_\mu=\mu^{\mathrm{c}}+2\mu^{\mathrm{m}}\,,\ \Delta_\mu=\mu^{\mathrm{c}}-\mu^{\mathrm{m}},
\end{aligned}\tag{17}$$

where the upper indices $^{\mathrm{c}}$ and $^{\mathrm{m}}$ denote the the core and the mantle material and r is the volume fraction of the cores in the core-mantle particles.

With these corrected formulae it turns out that the best measure for the importance of the induced magnetic dipole effects is not their contribution to the absorption efficiency but their contribution to the scattering amplitude, expressed by the ratio b_1/a_1.

$$\frac{b_1}{a_1}\approx\frac{1}{30}\,|\epsilon|\,x^2 \tag{18}$$

This ratio has to be compared with the relative error that one allows in a computation.

Fig. 2 shows the results of the application of the revised formulae to the optical behaviour of the dirty silicate grains with inclusions of iron particles. The embedded iron grains have radii of 7 nm, 60 nm and 120 nm and a volume concentration of 5 %. Figs. 2a and 2b contain the spectra of the absolute values of the dielectric functions and of the relative permeabilities for the three effective media. In Fig. 2c the absorption efficiency for small grains consisting of the effective materials are shown.

Now, the dielectric functions are the same for all cases, since they decouple from the magnetic behaviour.

In case of the 120-nm iron grains the small-x limit is already violated at the shortest wavelengths explaining the bending of the absorption curve below 1.6 μm.

In Fig. 3 we plotted the quantity $1/30\,|\epsilon|\,x^2$ from Eq. (18) as the better estimate for the importance of the magnetic dipole term for iron grains with the three considered radii.

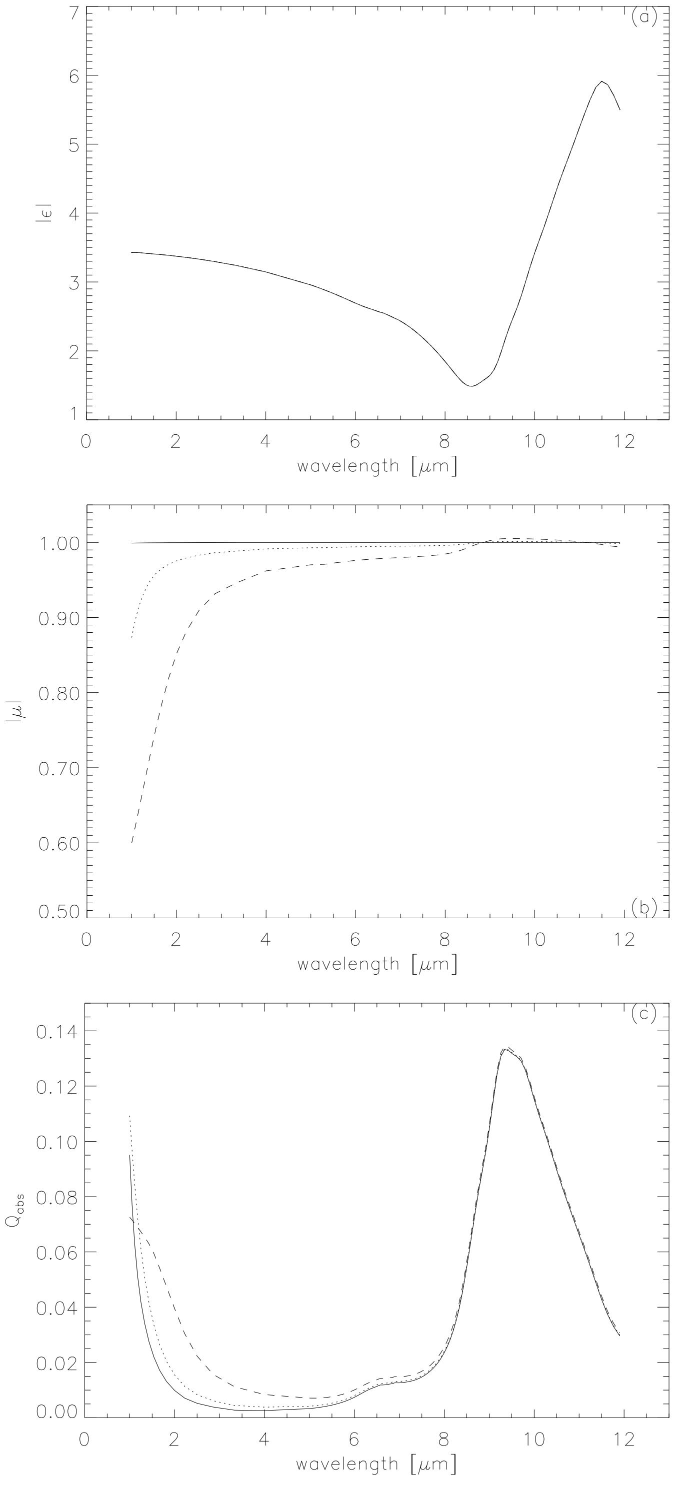

Fig. 2. Silicate with inclusion of iron grains with a radius of 7 nm (solid line), 60 nm (dotted line), and 120 nm (dashed line) : (a) absolute values of the effective dielectric functions; (b) absolute values of the effective permeabilities; (c) absorption efficiency for grains with a radius of 100 nm made of materials with the effective constants.

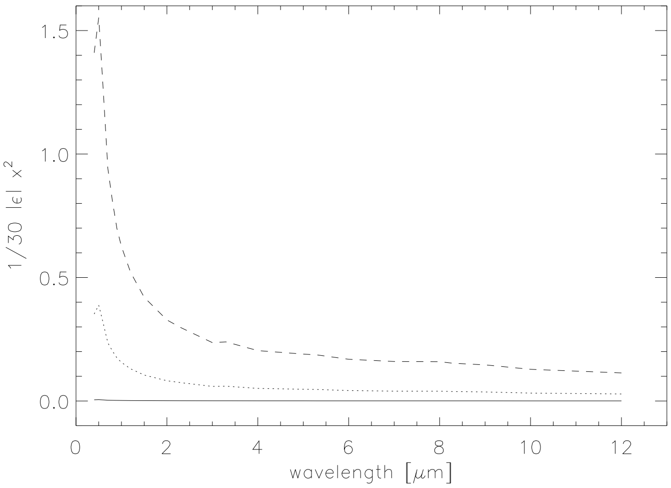

Fig. 3. Estimate of the relation between magnetic and electric scattering by the term $1/30\,|\epsilon|\,x^2$ for iron grains with a radius of 7 nm (solid line), 60 nm (dotted line), and 120 nm (dashed line).

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