Galactic Center problem sheet

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Stars around Sgr A*. Several bright stars of spectral type B are observed to be bound to the dark mass in the center of our Galaxy, so-called S-stars, see Fig. 1. To the first approximation their orbit is elliptical and can be treated in the framework of Newtonian gravity. The first star observed to complete its orbit was S2 (Schödel et al., 2002; Eckart et al., 2005). Based on its measured proper motion and acceleration it was possible to determine all six orbital elements of the elliptic orbit $(a, e, i, \omega, \Omega, t_{\rm P})$.

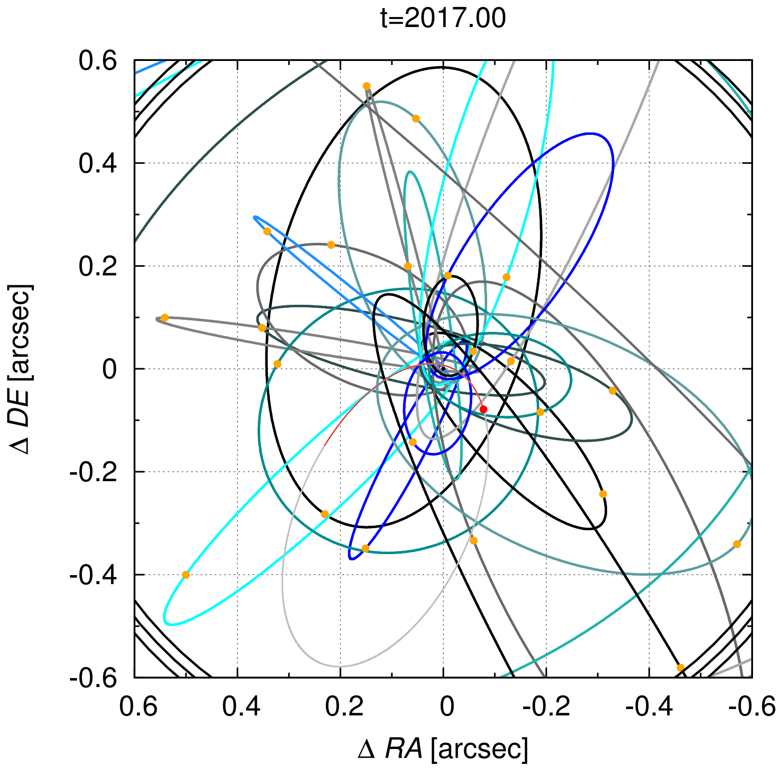

Figure 1: Projected orbits of the brightest S-stars. The Galactic center is at $(0, 0)$ position. The orbital elements used for plotting the orbits were taken from Gillessen et al. (2009); Valencia-S. et al. (2015); Meyer et al. (2012).

Try to calculate the following:

- The period of S2 star is $P \approx 15.8\,$yr, its semi-major axis in milliarcsecond $a \approx 123\,$mas, and its orbital eccentricity $e = 0.88$. What is approximately the **mass of the central dark object**? (for the conversion of arcsec to length units use the conversion relation 1arcsec $\approx 0.04\,$pc for the estimated distance of 8 kpc between the Sun and the Galactic centre). Given the calculated mass can you think of any stable configuration of the central dark mass other than the black hole? For the calculation you will need the gravitational constant $G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$ and the conversions: $1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m}$, $1\,\mathrm{yr} = 3.156 \times 10^7\,\mathrm{s}$, $1\,M_\odot = 2 \times 10^{30}\,\mathrm{kg}$, $c = 3 \times 10^8\,\mathrm{ms^{-1}}$.

- The most probable character of the compact radio source Sgr A* is the black hole. Black holes are quite simple objects – they are characterized by three parameters: mass, spin, and charge, which practically disappears due to neutralization. In case the black hole associated with Sgr A* does not rotate, the space-time structure around it is described by the spherically symmetric, vacuum solution of Einstein field equations:

$$\mathrm{d}s^2 = -(1 - \frac{r_\mathrm{s}}{r})c^2\mathrm{d}t^2 + (1 - \frac{r_\mathrm{s}}{r})^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\,, \tag{1}$$

  where $r_\mathrm{s} = 2GM_\bullet/c^2$ is the Schwarzschild radius and it is the radius of the sphere around the mass $M_\bullet$ from which the escape velocity is equal to the speed of light. This null hypersurface is characterized by the infinite redshift for the observer at infinity and is commonly referred to as the **event horizon**.

  Calculate the Schwarzschild **radius of the black hole** for the mass calculated in the previous problem. What is the distance of S2 star at the pericentre expressed in terms of Schwarzschild radii?

- The emission from Sgr A* becomes optically thin (we can see through) for the wavelengths of about 1 mm. Calculate the **necessary baseline of interferometric device** in order to resolve this emission on the scale of the event horizon of the Galactic center black hole.

- Estimate the **average density of the Galactic center black hole**. Compare it with the water density.

- There are about 100 young massive stars of OB spectral type (many of them Wolf-Rayet stars) within innermost parsec of the Galactic center. Due to strong winds their mas loss rate is about $\dot{M} = 10^{-5}\,M_\odot\mathrm{yr}^{-1}$. In case this material falls in from the distance of $\sim 0.05\,\mathrm{pc}$, calculate the **bolometric luminosity** produced by the accretion of this material onto the black hole in the Galactic center.

References

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