A statistical method to study turbulence in magnetized astrophysical environments

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Background

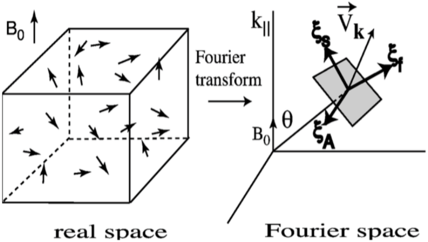
- Turbulence : stochastic but correlations are predictable.
- Turbulence statistics are anisotropic if **B** field is present (Goldreich & Sridhar 1995).
- Axisymmetric at best.
- Symmetry axis associated with direction of mean magnetic field.

Problem statement

- What parameters that characterise MHD turbulence can be studied statistically?
- Past analytical studies: e.g. Synchrotron fluctuations (Lazarian & Pogosyan 2012).
- Using spectroscopic data $\rightarrow$ new analytical study.

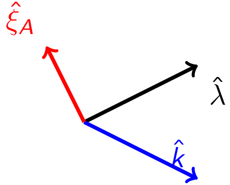
MHD approximation

- MHD turbulence $\rightarrow$ cascades of Alfvén, fast and slow modes (Cho & Lazarian 2002).
- Shown to only weakly exchange energy with each other (Cho & Lazarian 2002).
- Form approximately independent cascade.



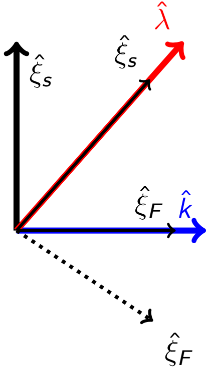
Cho & Lazarian, 2002

MHD modes



Alfvén mode

- Displacement $\hat{\xi}_A$ perpendicular to $\hat{\lambda} - \hat{k}$ plane



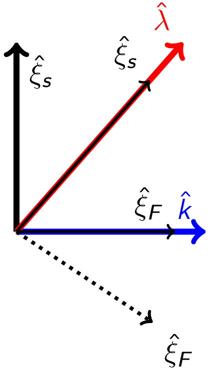
Fast mode

- Compressible mode
- Displacement $\hat{\xi}_F$ in the $\hat{\lambda} - \hat{k}$ plane
- For $\beta \gg 1$, $\hat{\xi}_F \parallel \hat{k}$, i.e. purely compressional
- For $\beta \ll 1$, $\hat{\xi}_F \perp \hat{\lambda}$

MHD modes

Slow mode

- Perpendicular to both Alfvén and fast modes
- For $\beta \gg 1$, displacement $\hat{\xi}_s \perp \hat{k}$
- For $\beta \ll 1$, $\hat{\xi}_s \parallel \hat{\lambda}$

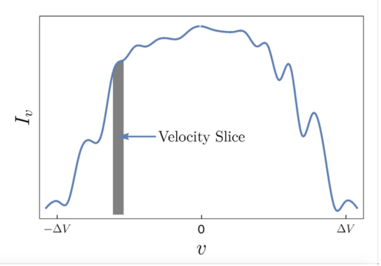


Significance of each MHD modes

- Alfvén modes : superdiffusion of cosmic rays (Lazarian & Yan 2014).
- Fast modes : cosmic ray acceleration (Yan & Lazarian 2012).

What we observe?

- emission lines
- e.g. 21 cm HI line
- Line profile sensitive to thermal and *turbulent* motions
- Correlate intensity along different LOS. (VCA, Lazarian & Pogosyan 2000)
- Correlate centroids along different LOS.



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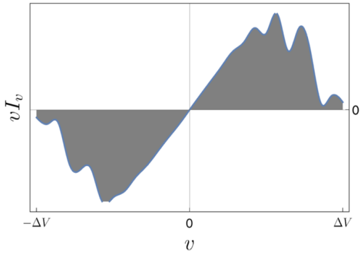
Velocity Centroids

- First moment of intensity

$$C(X) = \int_{-\infty}^{\infty} \mathrm{d}v \; v I_v(X) \; .$$

- Correlate centroids along different LOS.



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Statistical measure

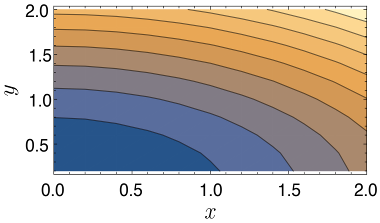
- Structure function $\mathcal{D}(\mathbf{R}) = \left\langle (\mathbf{C}(\mathbf{X_1}) - \mathbf{C}(\mathbf{X_1} + \mathbf{R}))^2 \right\rangle$.

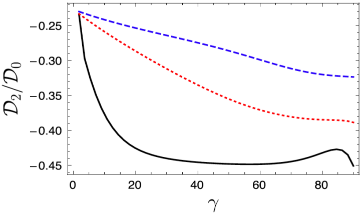
- For magnetised turbulent gas $\mathcal{D}$ is dependent on the sky angle (ϕ)
$$\mathcal{D}(R, \phi) = \sum_{n=-\infty}^{\infty} D_n(R) e^{in\phi} \ .$$

- We derived analytical expression for $D_n(R)$ (depends on specific MHD mode).

Results

- Alfvén modes: correlations elongated along the direction of projected **B** field
- i.e quadrupole to monopole is negative





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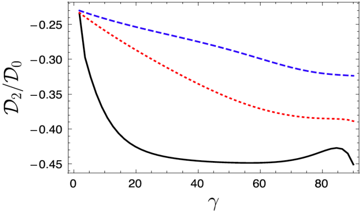
Results

- Quadrupole to monopole ratio increases for decreasing M_A.



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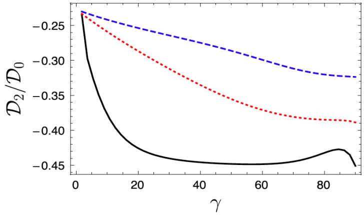
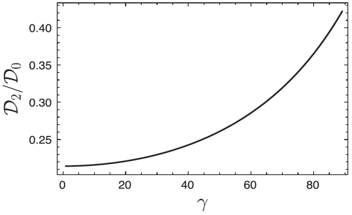
Results

- Fast modes have opposite quadrupole moment.
- Decrease in quadrupole to monopole implies more contribution of fast modes.





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Summary

- We developed analytical framework to study turbulence anisotropy using centroids.
- Anisotropy of turbulence leads to strong quadrupole moment in the structure function.
- Quadrupole to monopole ratio is observable and can be easily connected to theory
- Correlations carry important information about MHD modes, and the extent of compressibility.