New Diagnostics of MHD Turbulence
And What We Can Learn from Them!

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With David Collins, Alyssa Goodman, Chat Hull, Alex Lazarian, & Philip Mocz
What is Turbulence?

Turbulence is not just ‘chaos’. Turbulence is an energy transfer in space/time. It has specific statistical properties which can be seen when averaged over space/time.

\[ P(\bar{k}) = \sum_{\bar{k}=\text{const.}} \tilde{F}(\bar{k}) \cdot \tilde{F}^*(\bar{k}) \]

Three scales of interest in the energy cascade:

\[ P(k) \propto k^{-5/3} \]

"Big whorls have little whorls That feed on their velocity, And little whorls have lesser whorls And so on to viscosity"

Lewis Fry Richardson (1920)
Three basic parameters for simulations of ISM MHD turbulence

Outline for talk: Obtain these three parameters in the ISM

- Power spectrum (sources and sinks of energy)
- Sonic Mach number (ratio of turb./thermal energy)
- Alfvén Mach number (ratio of turb./magnetic energy)
- Gravity (important for GMC)
- Radiation/chemistry etc.

\[ E(k) \quad M_A = \frac{V}{V_A} \quad M_s = \frac{V}{c_s} \]

\[ u_A = \frac{B}{\sqrt{4\pi \rho}} \]

\[ \beta \equiv \frac{P}{P_B} \sim (M_A/M_s)^2 \]

\[ c_s = \sqrt{\frac{\gamma p}{\rho}} \]

Vazquez, Padoan, Passon, Stone, Mac Low, Klessen, Ostriker, Heitsch, Cho, Boldyrev, Li, Haugen, Jappsen, Ballestreros, Mee, Brandenburg, Kritsuk, Dib, Offner, Kowal, Schmidt, Cho, Lemaster, Glover, Federrath, Price, DelSordo, Collins, Hopkins, Walch, Robertson...++
Tool box for studying MHD turbulence:

Statistical Studies/Scaling Laws

Astrophysical Turbulence

Numerics: Synthetic observations

\[ E(k) \quad M_A = \frac{V}{V_A} \quad M_s = \frac{V}{c_s} \]
Turbulence Statistics and their Dependencies

- Probability Distribution Functions: Column Density, L. Pol
- Genus (topology): Column Density & L. Polarization
- Power Spectrum
- Bispectrum
- Tsallis Statistics
- Column Density

\( M_s \)
- Sonic Mach Number

\( M_A \)
- Alfven Mach Number

- VCS/VCA
- PPV
- Tree diagrams (dendrograms)

- HRO Column density polarization
- Velocity Anisotropy
- Velocity Centroid
Velocity/density power spectrum reveal multiphase ISM spectra in agreement with expectations for supersonic turbulence

For supersonic turbulence: density spectrum becomes shallower (less negative exponent) & velocity spectrum becomes steeper (more negative exponent) relative to Kolmogorov spectrum.

<table>
<thead>
<tr>
<th>N</th>
<th>data</th>
<th>Object</th>
<th>$P_{PPV}^{thin}$</th>
<th>$P_{PPV}^{thick}$</th>
<th>depth</th>
<th>$E_v$</th>
<th>$E_\rho$</th>
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<td>HI</td>
<td>Anticenter</td>
<td>$K^{-2.7}$</td>
<td>N/A</td>
<td>Thin</td>
<td>$k^{-1.7}$</td>
<td>N/A</td>
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<td>N/A</td>
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<td>$k^{-1.7}$</td>
<td>$k^{-1.4}$</td>
</tr>
<tr>
<td>4</td>
<td>HI</td>
<td>Center</td>
<td>$K^{-3}$</td>
<td>$K^{-3}$</td>
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<td>N/A</td>
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<tr>
<td>5</td>
<td>HI</td>
<td>B. Mag.</td>
<td>$K^{-2.6}$</td>
<td>$K^{-3.4}$</td>
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<td>$k^{-1.8}$</td>
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<tr>
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<td>HI</td>
<td>Arm</td>
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<tr>
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<td>HI</td>
<td>DDO 210</td>
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<td>$K^{-3}$</td>
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<td>N/A</td>
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<tr>
<td>8</td>
<td>12CO</td>
<td>L1512</td>
<td>N/A</td>
<td>$K^{-2.8}$</td>
<td>Thick</td>
<td>N/A</td>
<td>$k^{-0.8}$</td>
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<td>Perseus</td>
<td>$K^{-2.7}$</td>
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<td>$k^{-1.7}$</td>
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<tr>
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<td>$k^{-1.7}$</td>
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</table>

Compare to $-5/3=-1.66$

Integrated intensity has a slope of -3 in the optically thick limit (Burkhart et al. 2013).

SMC in 21 cm emission

Radio data is ideal for studies of turbulence because it contains information about turbulence velocity along the LOS.

Stanimirovic et al. 1999 data set has good spatial (98") and spectral resolution (1.65kms\(^{-1}\)) and contains both single dish (Parkes Telescope) and interferometer (ATCA telescope) data (30pc-4kpc).
Velocity Coordinate Spectrum (VCS) of SMC (in 21cm)

VCS technique developed in Lazarian & Pogosyan 2006, 2008

For application of velocity power spectrum to $^{13}$CO see poster S2-6 by Sac Nicte Medina

Model parameters:
- $\alpha_v = 3.85$, $L_v = 2300$ pc
- $F_{ij}(k) = \frac{V_0^2}{L \alpha_v} e^{-\frac{k^2}{L^2}}$
- $\alpha_v = 3.85$
- $\alpha_z = 3.04$ (Stanimirovic & Lazarian 2001)
- $L_v = 2300$ pc
- $v_{turb} = 6700$ m/s

VCS fitting for SMC
Effective beamwidths:
- $0^\circ.025$
- $0^\circ.050$
- $0^\circ.100$

Lines - model

Chepurnov, Burkhart, Lazarian & Stanimirovic 2015
PDFs of Column Density in the \textit{diffuse ISM}: $M_s = \frac{v}{c_s}$

2\textsuperscript{nd} moment: Variance ($\sigma^2$ linear and log PDF) vs. $M_s$
3\textsuperscript{rd} moment: Skewness (linear PDF) vs. $M_s$
4\textsuperscript{th} moment: Kurtosis (linear PDF) vs. $M_s$

Column density PDFs:
Kowal et al. 07; Burkhart et al. 09, 10; Burkhart & Lazarian 12; Federrath et al. 12, Kainulainen & Tan 13
The WNM/CNM ISM PDF: Sonic Mach Number vs. Variance

Black lines: prediction from Burkhart & Lazarian 2012
Black points: ideal MHD simulations.

(data from Berkhuijsen & Fletcher 2008)
Sonic Mach Number in CNM

Observational Method for Cold Neutral Mach Numbers (need spin temperature).

\[ M_s^2 = \frac{V_{t,3D}^2}{C_s^2} = 3.7 \left( \frac{T_{k,\text{max}}}{T_s} - 1 \right) \]

\[ \frac{N_1}{N_0} \equiv \frac{g_1}{g_0} \exp\left( -\frac{h\nu_{10}}{kT_s} \right) \]

Small Magellanic Cloud CNM Mach number

(spinn temperatures from Dickey et al. 2001)

CNM PDF prediction: \( M_s = 3-20 \)

Burkhart et al. 2010
PDFs of Magneto-gravo-turbulence

Power law tails observed in column density: Burkhart, Collins & Lazarian (2015)

“Low B”, $M_A=20$

Agrees well with Herschel observations of Schneider et al. 2014

Analytic expression for transition point presented in Burkhart, Stalpes & Collins 2016
Power Law Tail Slopes

Implication: PDF power law tails can be used to determine the evolutionary stage of the cloud!

Burkhart, Collins & Lazarian 2015
Multiphase View of Perseus: HI and H2

Total gas/dust (2MASS/IRAS)

Opacity-corrected N(HI) (Lee+15)

Re-derived N(H2) (following Lee+12)

\[
N(H_2) = \frac{1}{2} \left( \frac{A_V}{D/G} - N(HI) \right)
\]

\( (D/G) \) of \( \sim 1 \times 10^{-21} \)
Multiphase View of the Atomic and Molecular Gas PDF of Perseus

\[ \frac{\rho_{\text{crit}}}{\langle \rho \rangle} = \frac{1}{3} M_s^2 \]

Transition point roughly at post shock density as seen in simulations (Mocz et al. 2017) and analytically (Krumholz & McKee 2005; Burkhart, Stallpes Collins 2016)

Sharp HI-H2 transition traced by PDFs near Krumholz, McKee & Tumlinson (2009) predicted value (~10 Msolar/pc²). 

HI PDF traces lognormal (turbulence)

H₂ PDF traces power law tail (gravity)

Total gas PDF shows the composition of both.
PDFs of HI/Dust are needed for observational methods to find the HI to H2 transition in clouds when including realistic turbulent density fluctuations!

Bialy, Burkhart & Sternberg 2017
What about the magnetic field?

“I hate magnetic fields!” – anonymous astronomer

“Magnetic fields – ugh” – anonymous astronomer

“Magnetic fields are not important.” – anonymous astronomer

\[
M_A = \frac{V}{V_A} = \frac{B}{\sqrt{4\pi \rho}}
\]

\[
\beta \equiv \frac{P}{P_B} \sim \left( \frac{M_A}{M_s} \right)^2
\]

See talk by Lazarian on Tuesday for measuring Alfven Mach number with velocity gradients.
Measuring $M_A$: The magnetic Field is of critical importance for star formation


<table>
<thead>
<tr>
<th>sim.</th>
<th>$\beta_{\text{mean-field}}$</th>
<th>$B_{\text{mean-field}}$ ($\mu$G)</th>
<th>$M_{A,\text{mean-field}}$</th>
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<td>1.2</td>
<td>35</td>
<td>10</td>
<td>very weak field</td>
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<tr>
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<td>3</td>
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<td>36</td>
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<tr>
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<td>0.0025</td>
<td>120</td>
<td>0.35</td>
<td>10</td>
<td>strong field</td>
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</tbody>
</table>

We vary magnetic field strength in four different simulations

Mocz, Burkhart, Hernquist, McKee & Springel 2017
Mocz, Burkhart, Hernquist, McKee & Springel 2017
Sub- vs. super-Alfvenic collapse produces very different density/polarization signatures from 5pc to 100AU.
Magnetic field strength at large scales affects field alignment at small scales. Fraction of gas with the magnetic field aligned within 30 degrees of the large-scale mean-field value.

Magnetic field strength at pc scale strongly affects collapsing cores (hourglass, field alignment etc). ALMA will provide the ideal tool to study field structure at AU scales.

Mocz, Burkhart, Hernquist, McKee & Springel 2017
Alignment of field and density observed in polarization with ALMA

Histogram of Relative Orientation at ~100 AU scales

\[ \phi = \arctan \left( \frac{|B \times \nabla n|}{B \cdot \nabla n} \right) \]

\( M_A = 0.35 \)
\( 1.2 \)
\( 3.5 \)
\( 35 \)
\( \text{Data} \)

\( \leftarrow \) High magnetic field
\( \leftarrow \) Low magnetic field

Data: Ser-emb 8

See talk by Juan Soler for HRO for magnetic field studies at parsec scales!
Conclusions: We are in a new era of measurements of turbulence which will enable us to understand the life cycle of gas in galaxies and star formation.

1) Measuring the power spectrum: Diagnostics for studies of turbulence are able to obtain turbulence parameters. We can now measure the velocity power spectrum in the ISM using available spectral line survey data (21cm, CO, HCN, N2H+etc) with VCS. There is a lot of work ahead applying diagnostic tools to data sets!

2) Measuring the sonic Mach number: PDFs not only give information on $M_s$ but are also useful for describing collapse (power law tails) and HI-H2 transition (transition point).

3) Measuring the Alfvenic Mach number: The Magnetic field is critical for regulating the structure of collapsing clouds. Relative orientations of B and density from parsec to AU scales can indicate if magnetic energy dominates over kinetic energy.