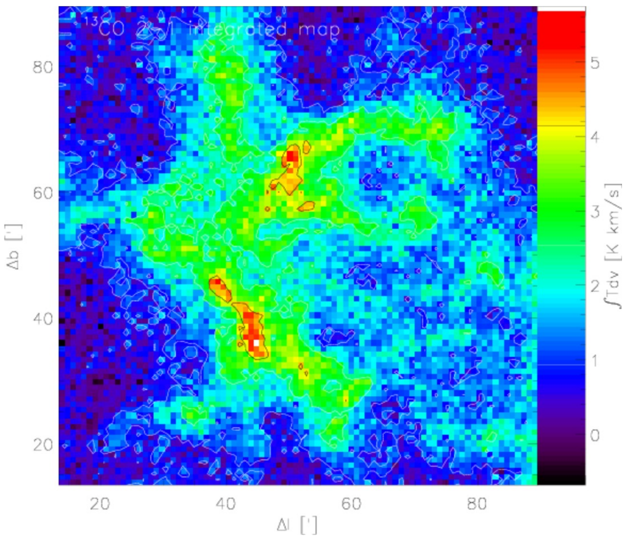

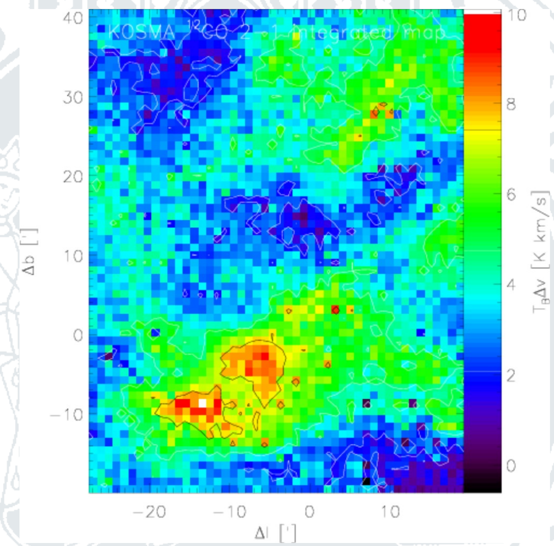

Wavelet approaches for measuring interstellar cloud structure

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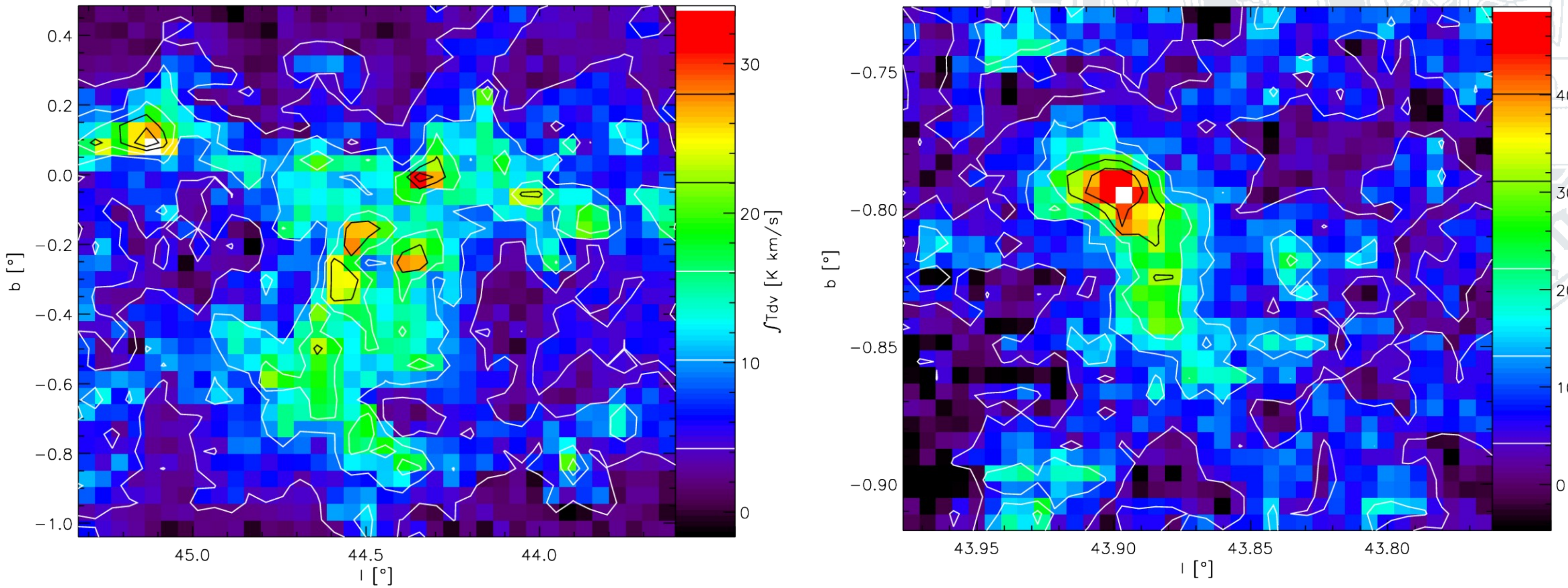
I. Physikalisches Institut, Universität zu Köln

Turbulent cascade

Self similarity

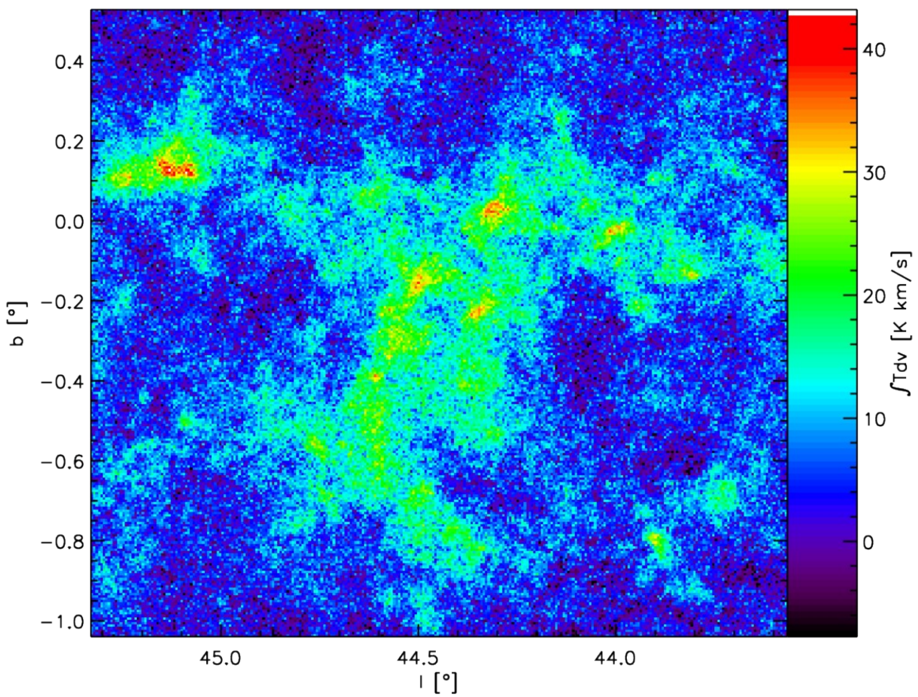
- **Same type of structures on all scales**
  - Confirmed by observations:

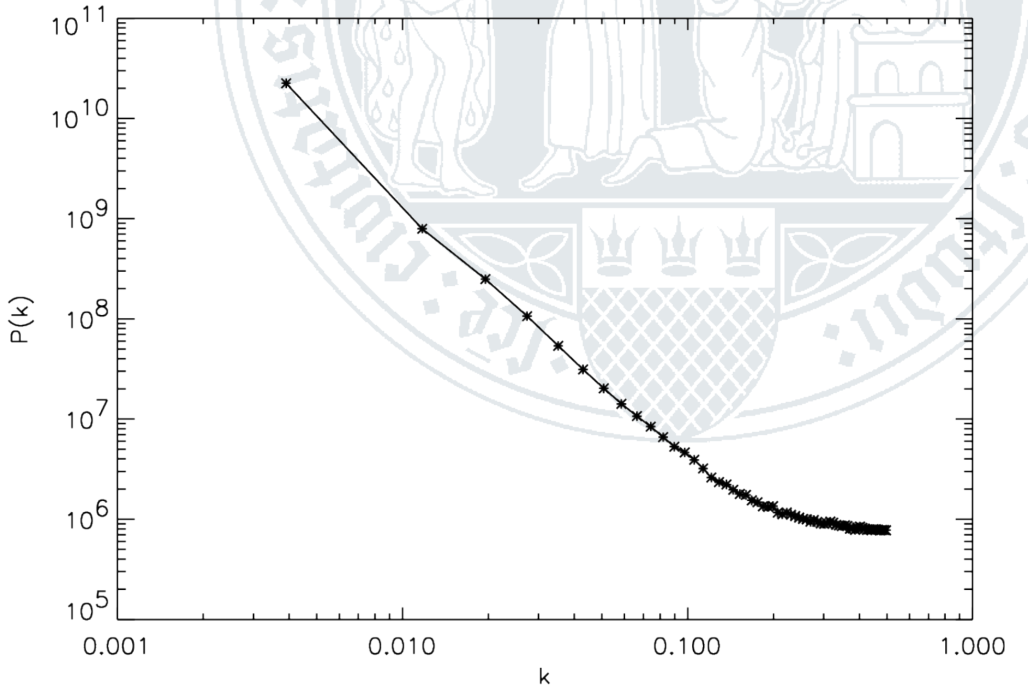


  - Integrated ^{13}CO 1-0 line map from the BU-FCRAO survey of the Galactic Ring at different “zoom levels” (Simon et al. 2002)
  - ☞ Self-similar over a large range of scales.

Power spectrum

Scale invariance results in a power-law power spectrum: $P(k) \propto k^{-\beta}$





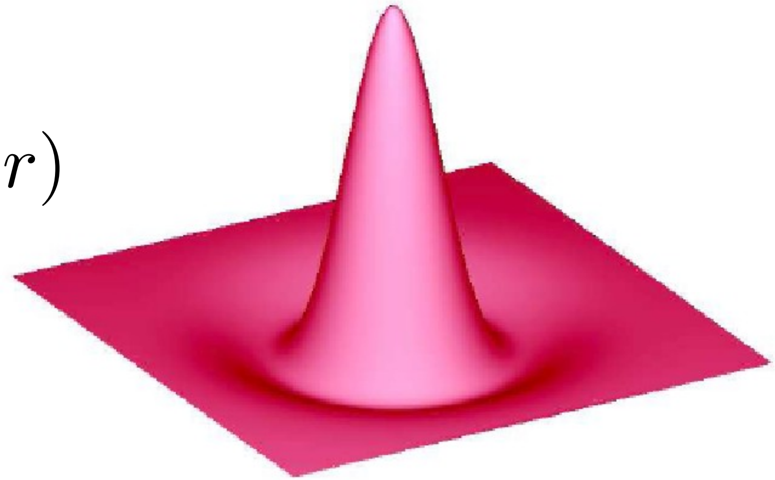
Analysis of G44.5 subfield of ^{13}CO 1-0 line map from BU-FCRAO-GRS

- For rectangular maps, the power spectrum can be easily computed by FFT.

Measure spatial scaling

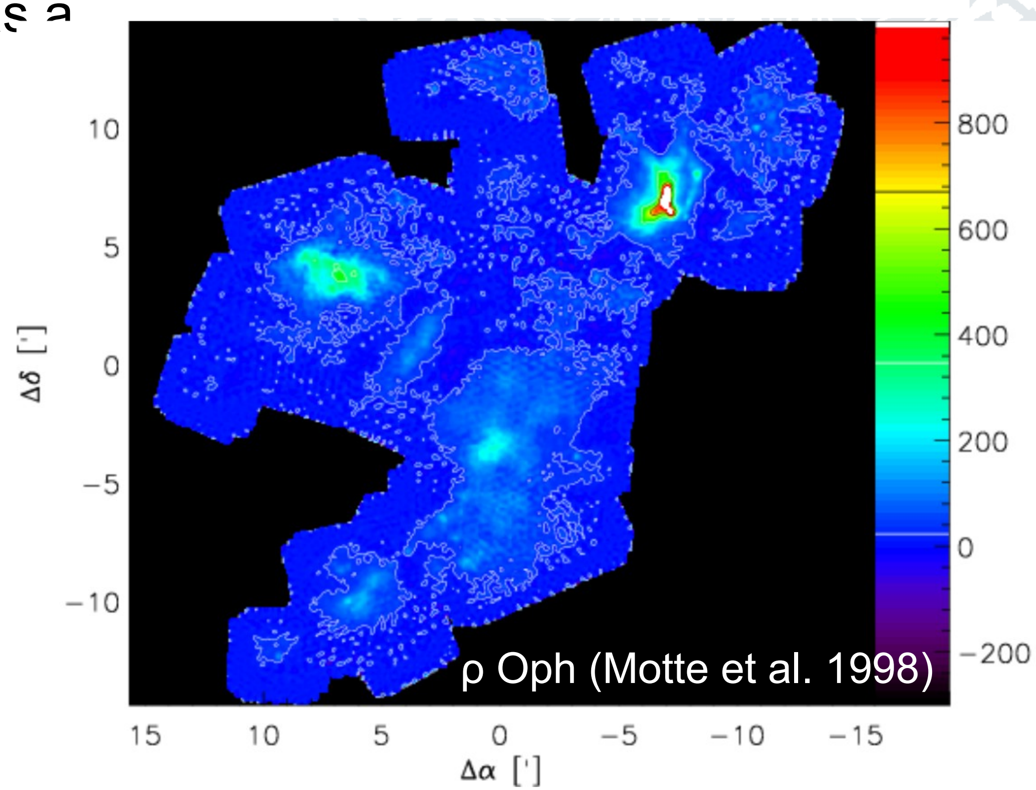
Δ-variance

Filter map by radially symmetric wavelet $\psi_l(r)$



- characteristic length scale *l*
- Measure variance in convolved image as function of the filter size *l*
- Gives relative amount of structure as a function of structure size

- **Advantages compared to power spectrum**
  - Δ-variance can measure spatial scaling of irregular maps, maps with variable noise.
  - In the application to observed maps the Δ-variance is much more robust.



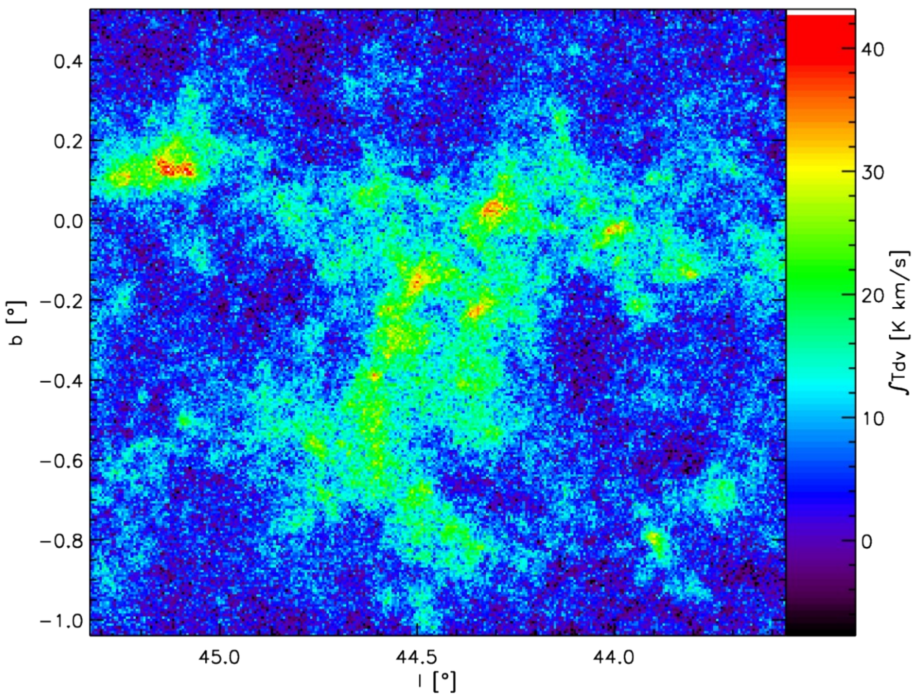
ρ Oph (Motte et al. 1998)

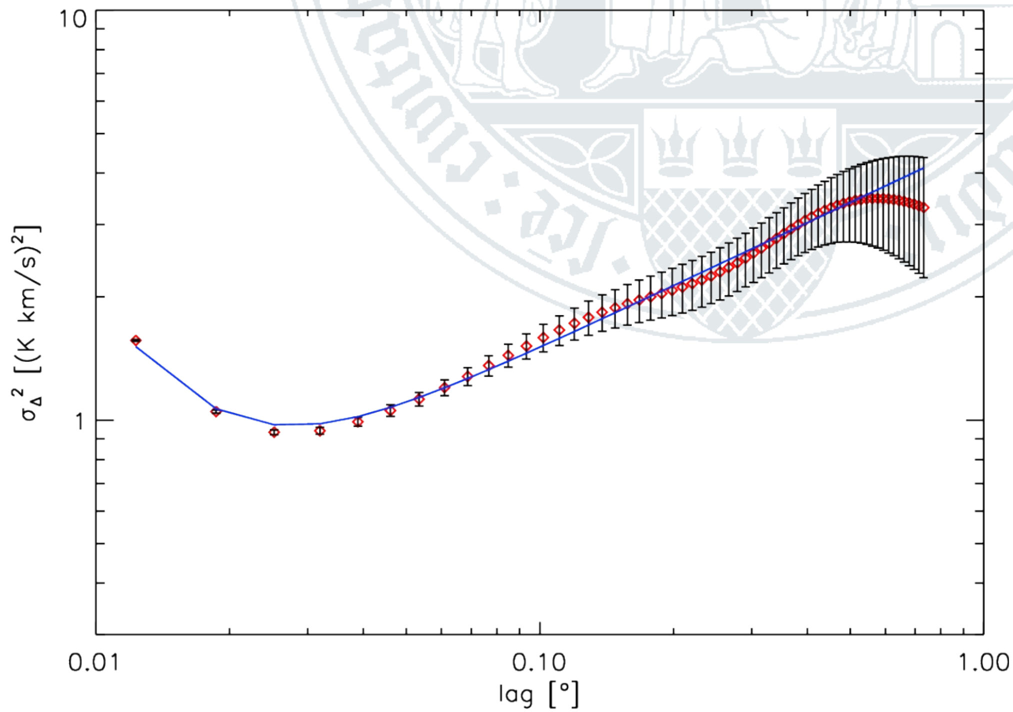
Measure spatial scaling

Δ-variance

Power-law power spectrum gives power-law Δ-variance: $\sigma_\Delta^2(l) \propto l^\alpha$

- Spectral index related to the power spectral index by $\alpha = \beta - 2$.



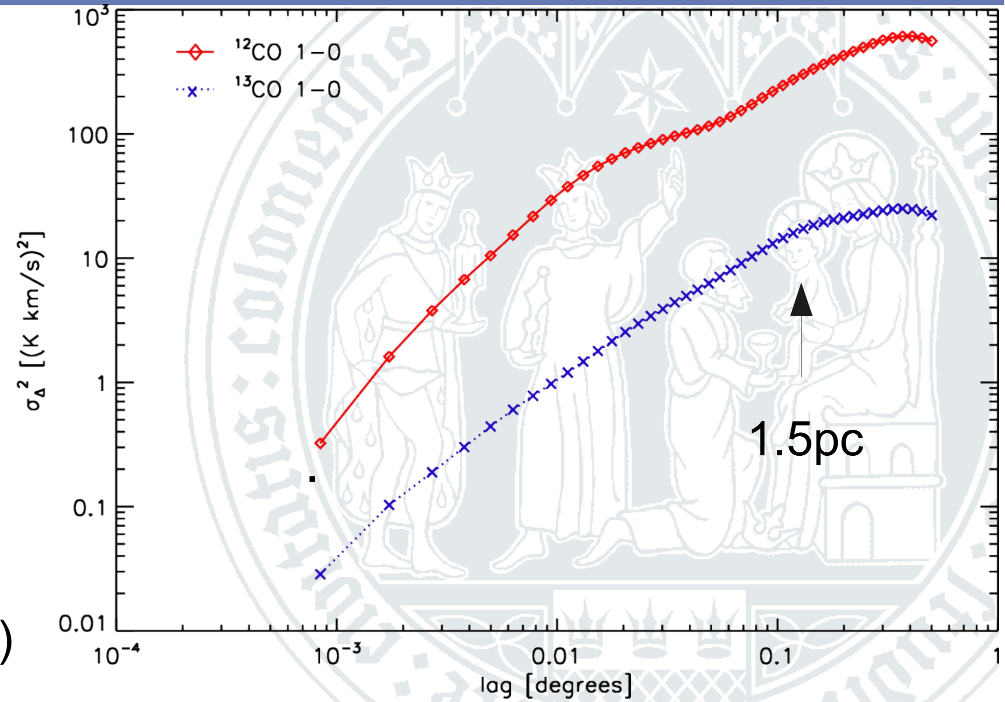


Analysis of G44.5 subfield of ^{13}CO 1-0 line map from BU-FCRAO-GRS

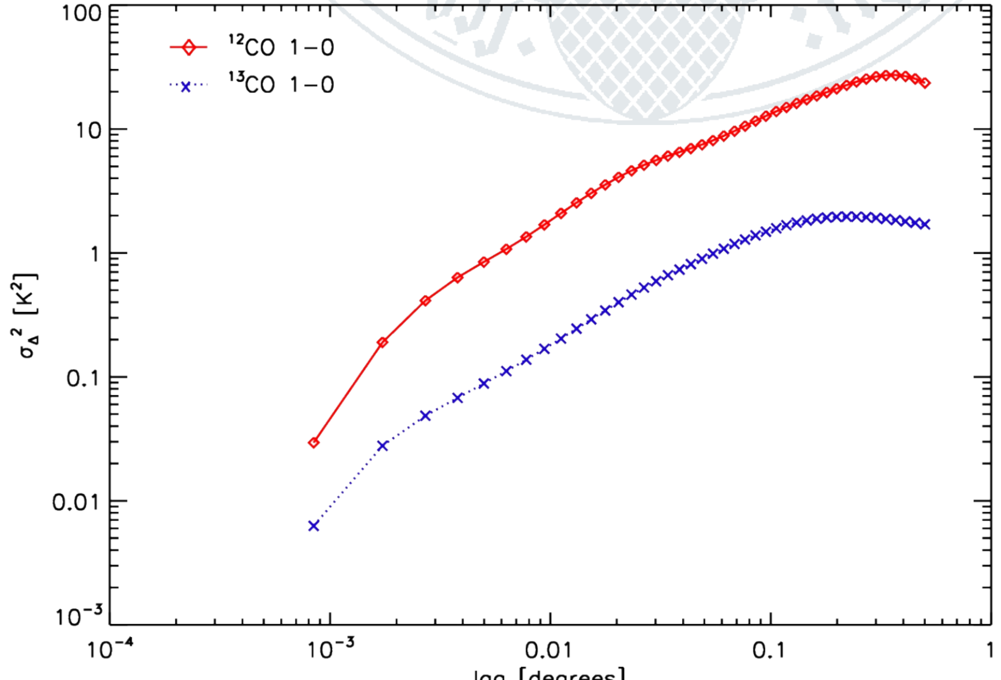
Analysis of Orion Bar maps

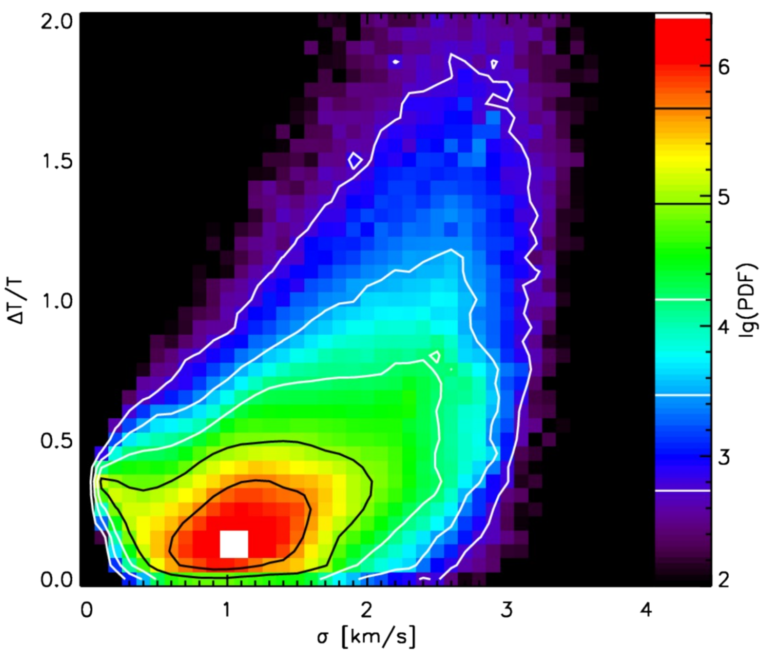
Δ-variance spectra

- No artifacts from combination of single dish and interferometer or mosaicing
- Perfect power law for ^{13}CO, $\beta=3.1$
- Some characteristic scales in all spectra



Δ-variance spectra of integrated maps (top) and 8.8km/s channel maps (bottom, Acre et al. in prep)





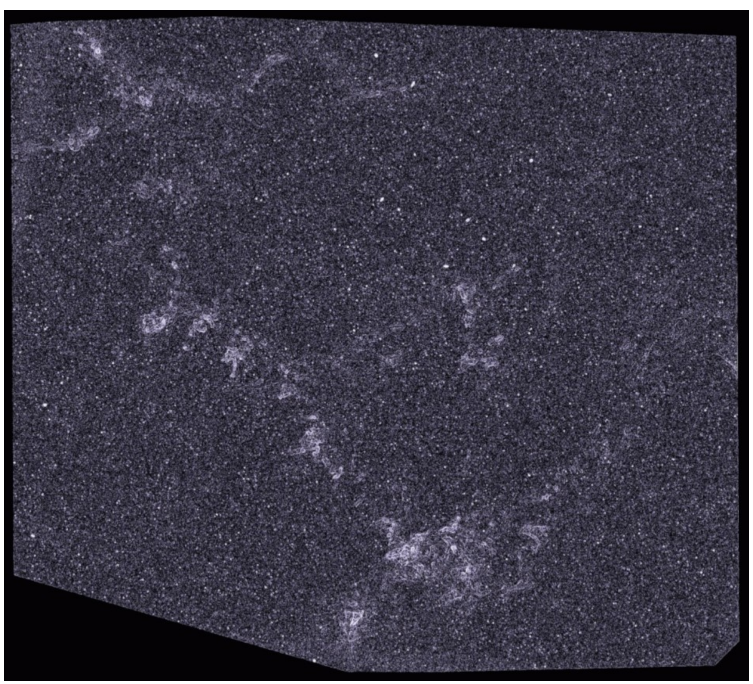
"Tauber"-line width plot for ^{13}CO

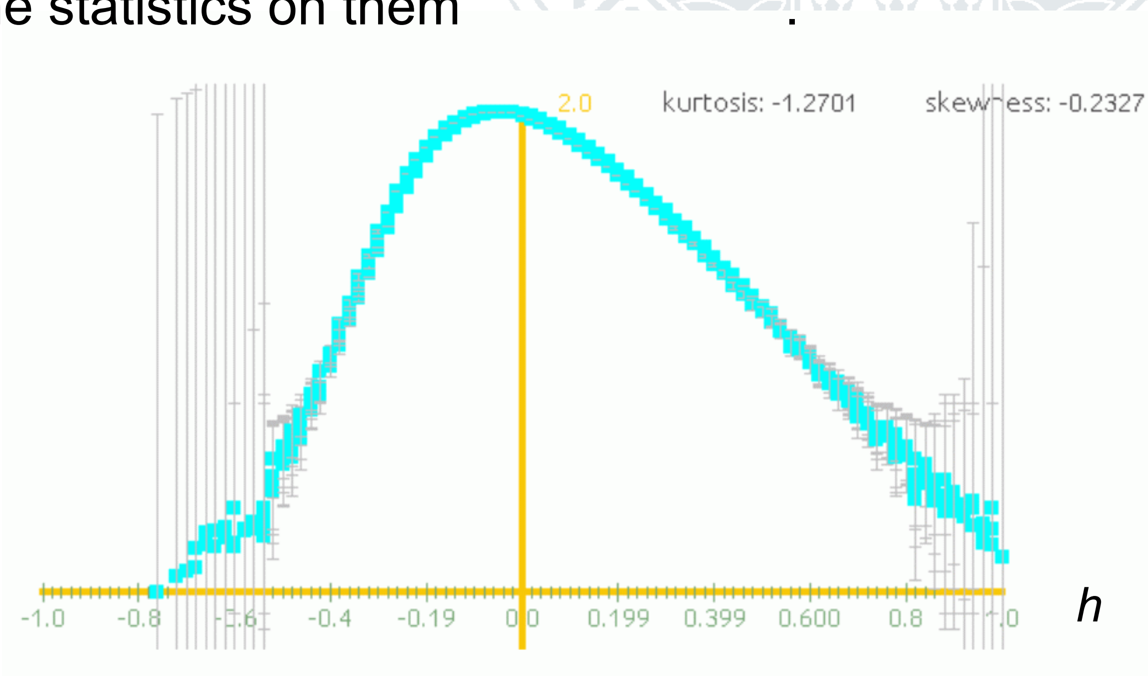
Use for singularity spectrum

INRIA approach (Hussein Yahia)

Spatial distribution of box-car dimension at small scales

- *h(x)* is scaling exponent for 3-D "ball" around point x with radius r
- For $r \rightarrow 0$ (smallest resolution available), scales as $r^{h(x)} \rightarrow$ *h(x)* is the singularity exponent at *x*.
- Provide precise evaluation of the strength of fronts observables in clouds, and the possibility of computing fine statistics on them .



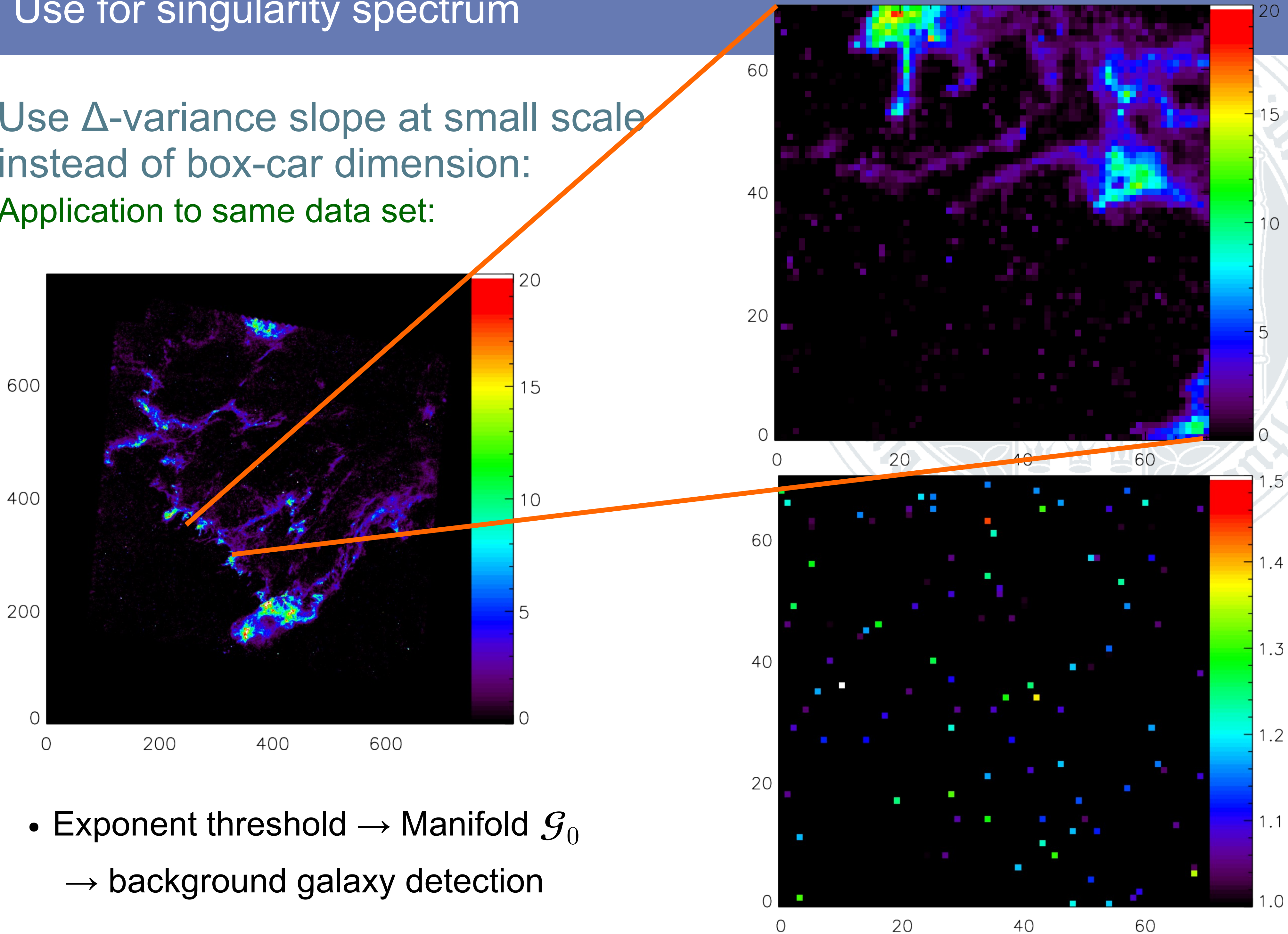


Singularity distribution for Draco 250 μm map

Use for singularity spectrum

Use Δ-variance slope at small scale instead of box-car dimension:

Application to same data set:



- Exponent threshold → Manifold $\mathcal{G}_0$

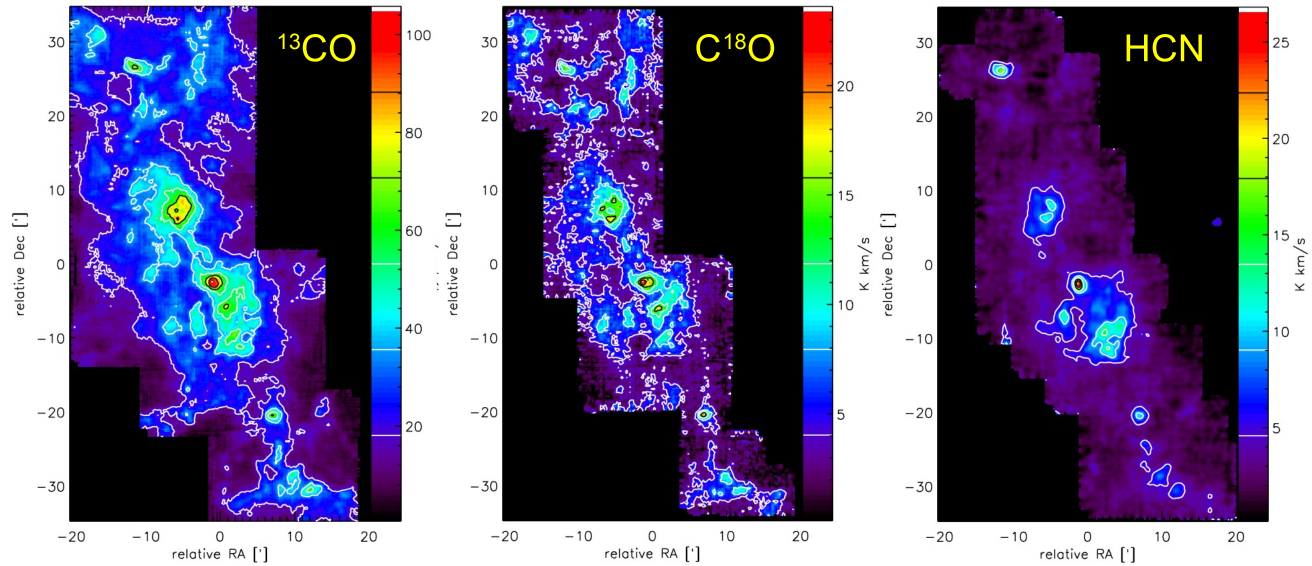
  → background galaxy detection

Wavelet-weighted cross-correlation analysis (WWCC)

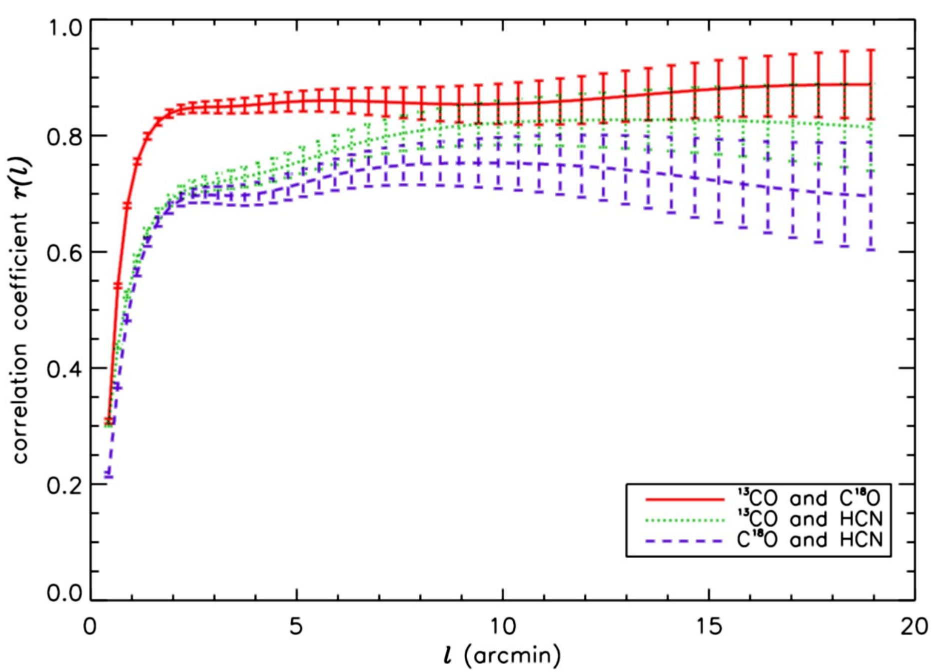
WWCC:

- Cross correlation in maps filtered by Δ-variance wavelet
- compare map structures depending on their scale

Apply to line observations of G333 (MOPRA):



(Arshakian & Ossenkopf 2015)



Result:

- ^{13}CO-C^{18}O perfectly correlated above the noise scale
- HCN behaves different at scales <7´ (~8pc)

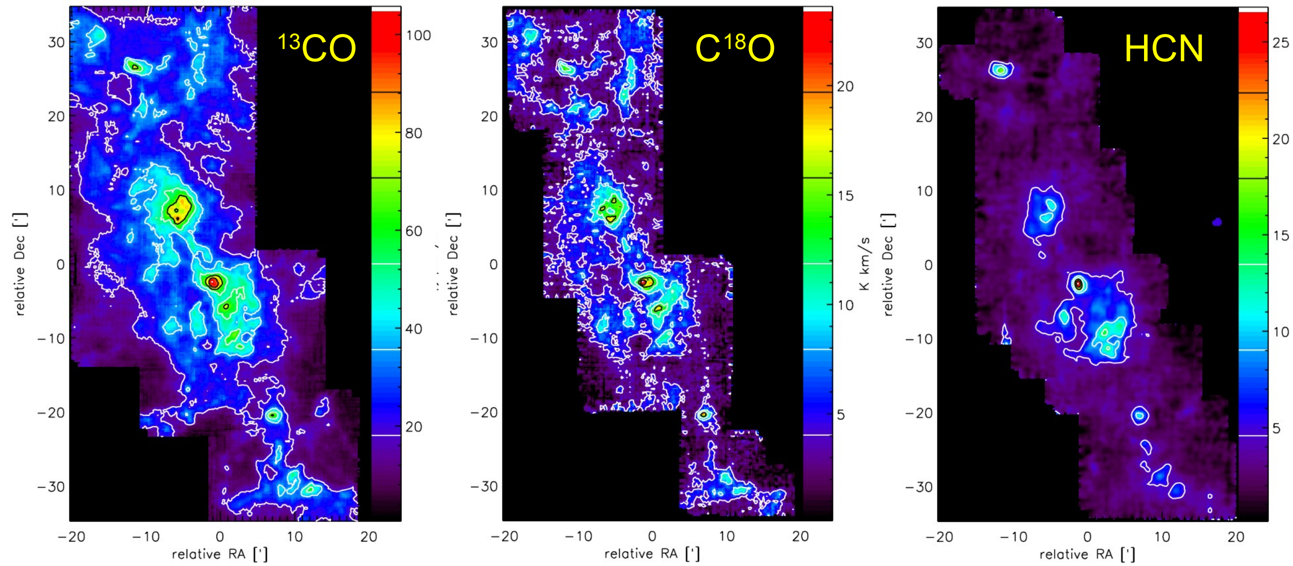
Difference in chemical structure or excitation conditions?

Wavelet-weighted cross-correlation analysis (WWCC)

WWCC:

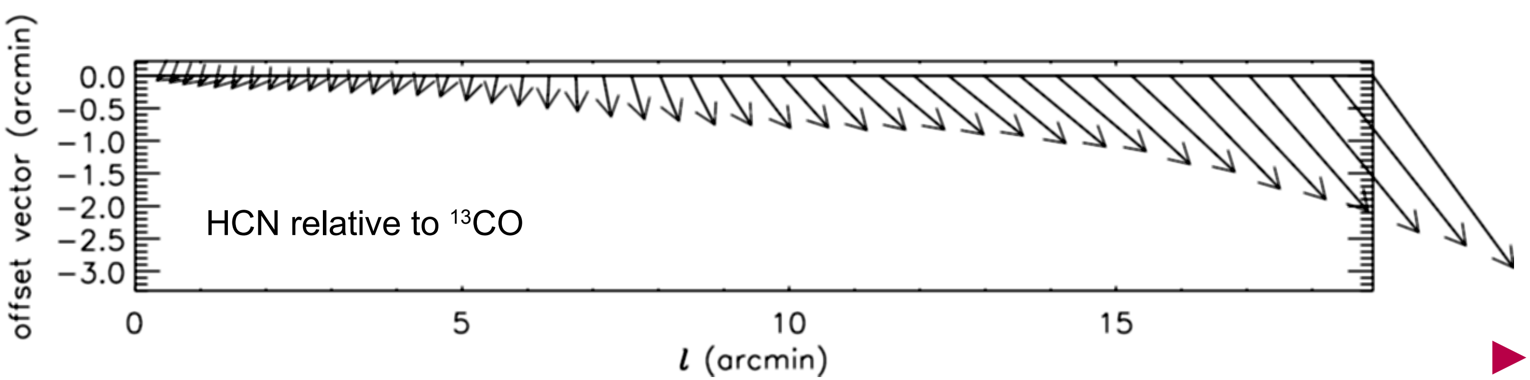
- Cross correlation in maps filtered by Δ-variance wavelet
- compare map structures depending on their scale

Apply to line observations of G333 (MOPRA):



(Arshakian & Ossenkopf 2015)

Scale-dependent offsets:



- Increasing offset along the filament towards larger scales

► cometary density structure or/and anisotropic radiation field

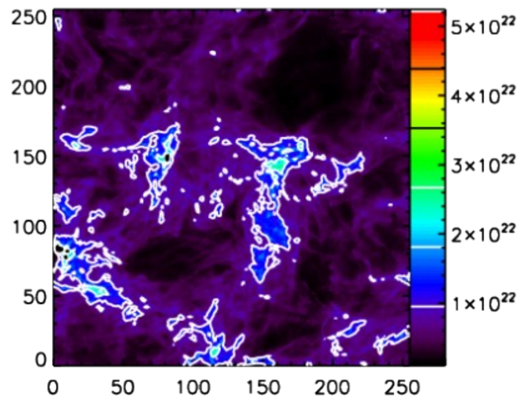
Simulations: (M)HD models from Federrath & Klessen (2012)

Compute line radiative transfer
(Simline3D, Ossenkopf 2002):

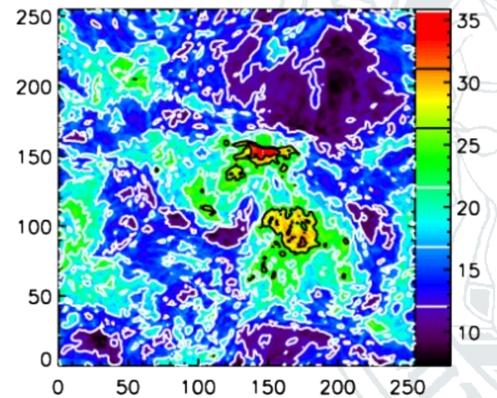
Integrated line intensities [K km/s]

Simplifications applied here:

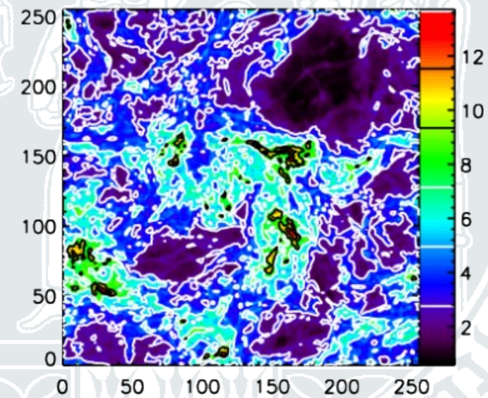
- Constant abundances
- Isothermal



Column density



CO 1-0



^{13}CO 1-0

FK12 Model 20:

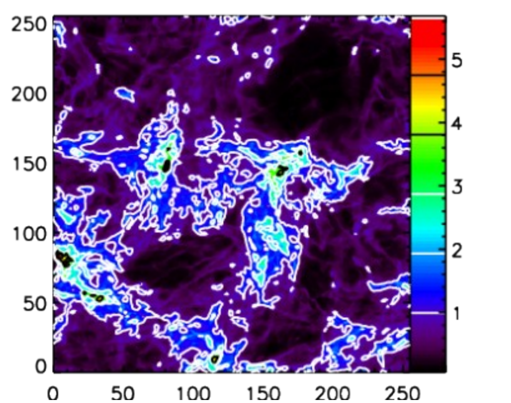
$M = 10\ M_\odot$

$D = 8\mathrm{pc}$

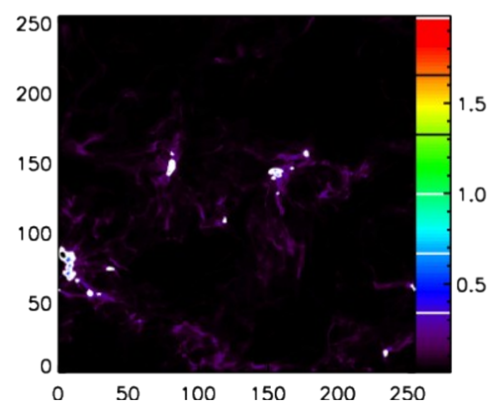
$<n> = 207\mathrm{cm}^{-3}$

$<B> = 3\mu\mathrm{G}$

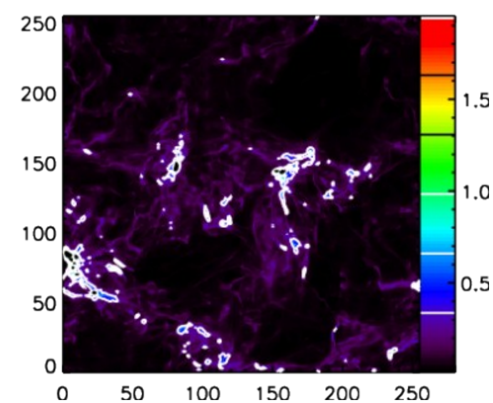
C^{18}O 1-0



CS 2-1

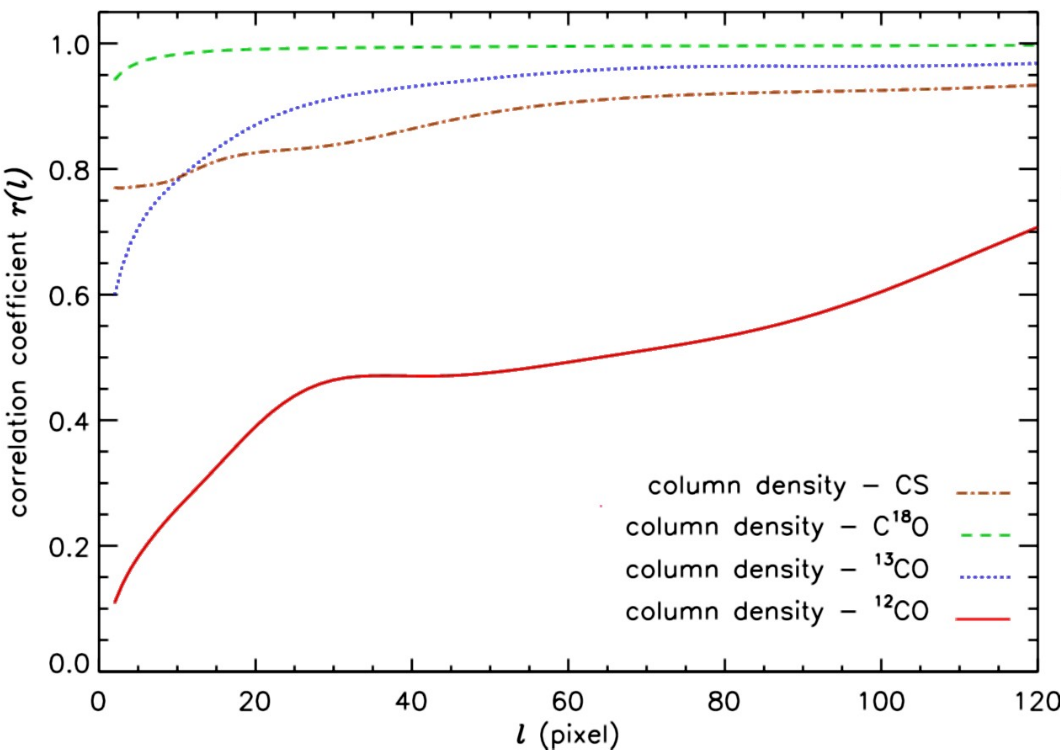


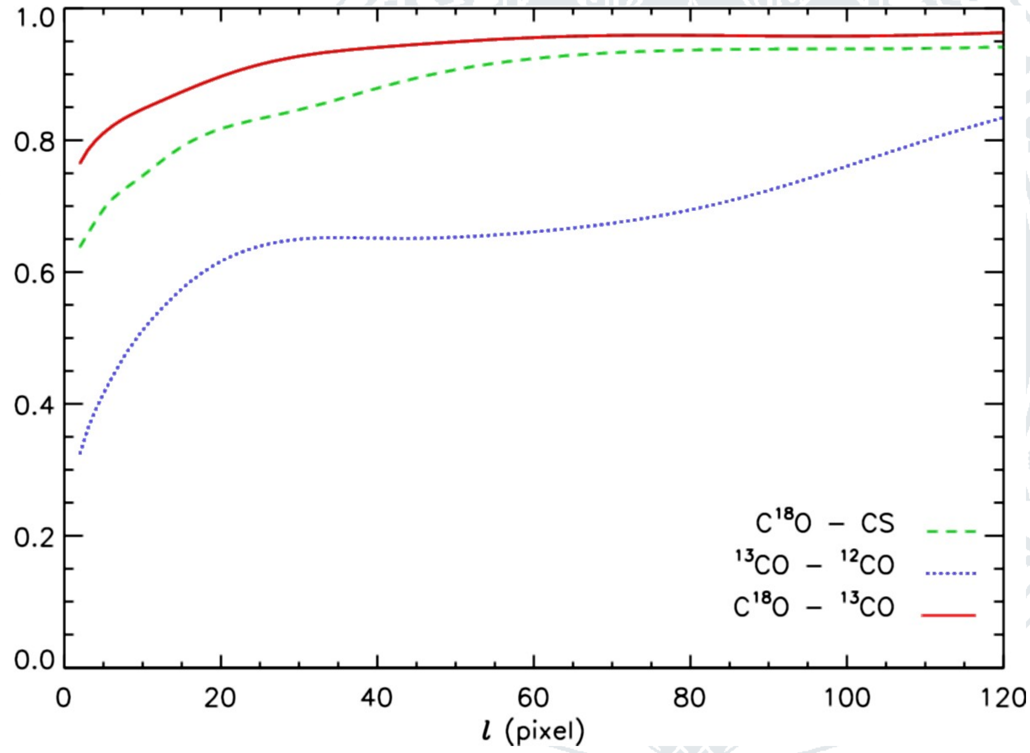
N_2H^+ 1-0



- No line traces true column density,
- Rare CO isotopes best approximation

Cross-correlation function





Results:

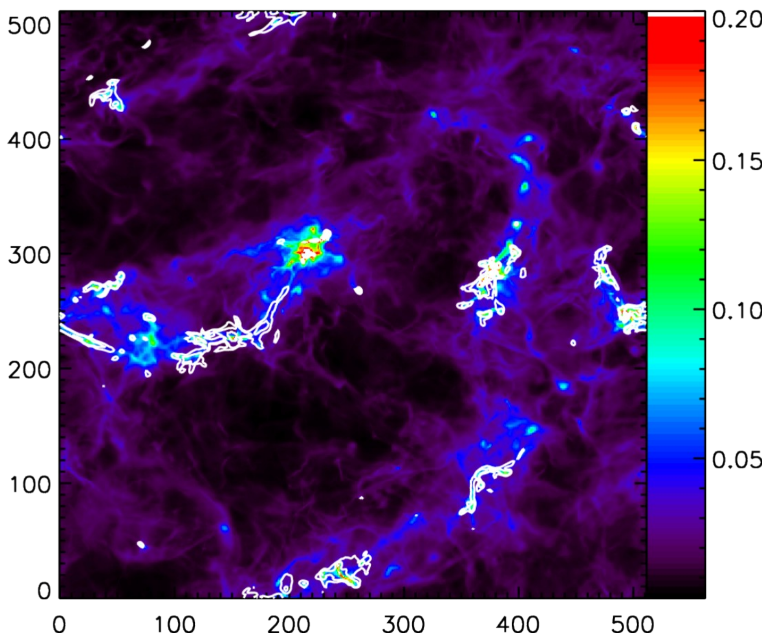
- Perfect correlation between spatial density structure and $C^{18}O$ emission
  - High correlation between $C^{18}O$ and ^{13}CO like in the observations
- Lower correlation for CS (like HCN in observations)
  - But no critical size scale
- ^{12}CO hardly reflects underlying density structure
  - only measures the velocity structure

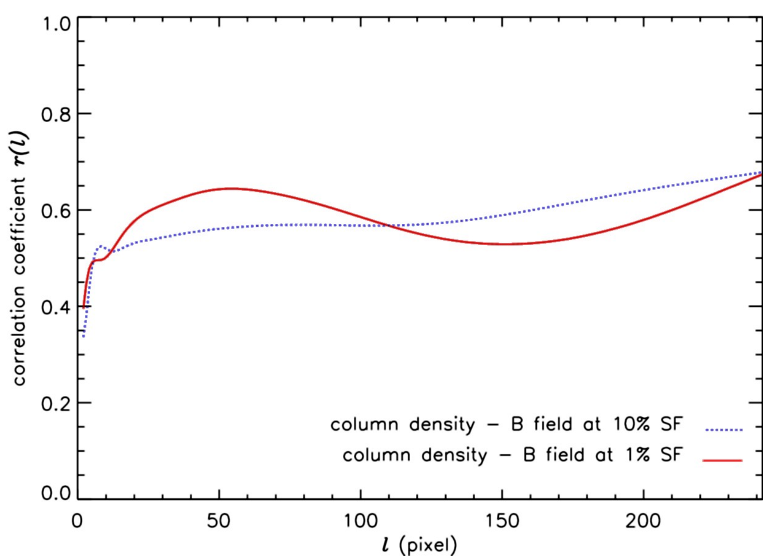
Cross-correlation function

Compare direct properties of simulations:

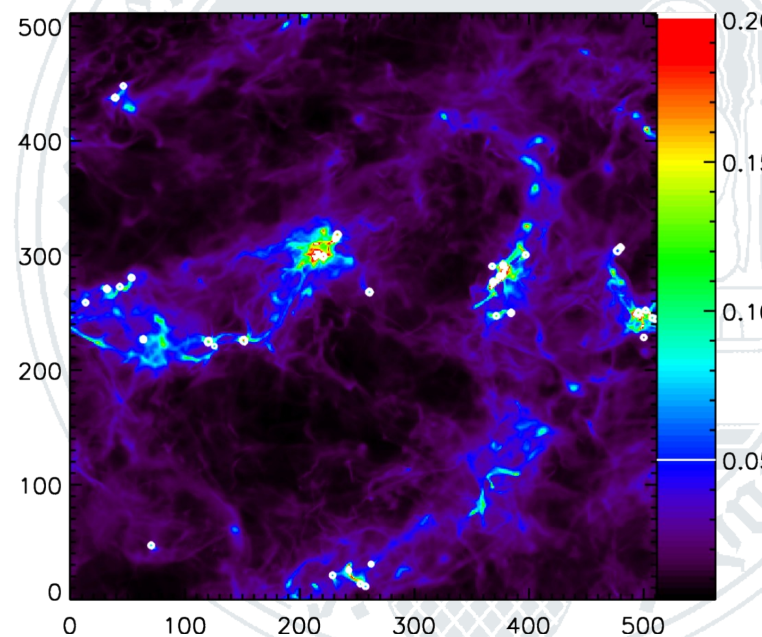
- Weak and constant correlation between density and B-field
- Growing scale of SF feeding with time

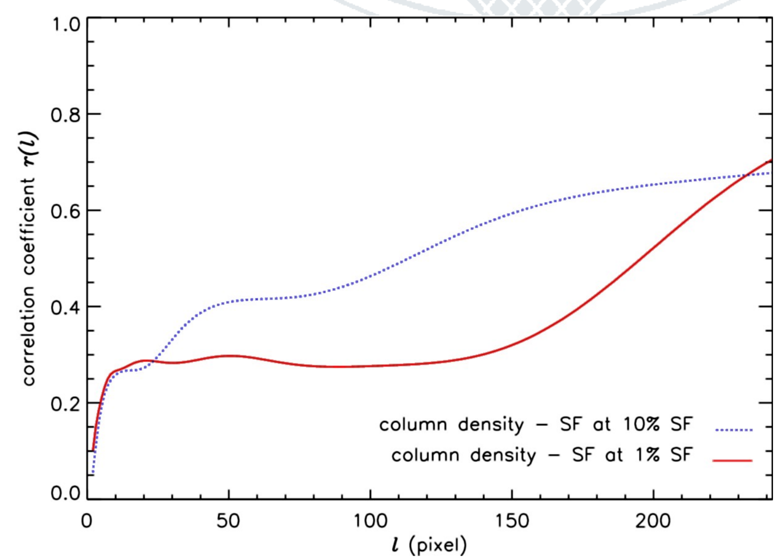
Column density – magnetic field





Column density – Protostellar cores



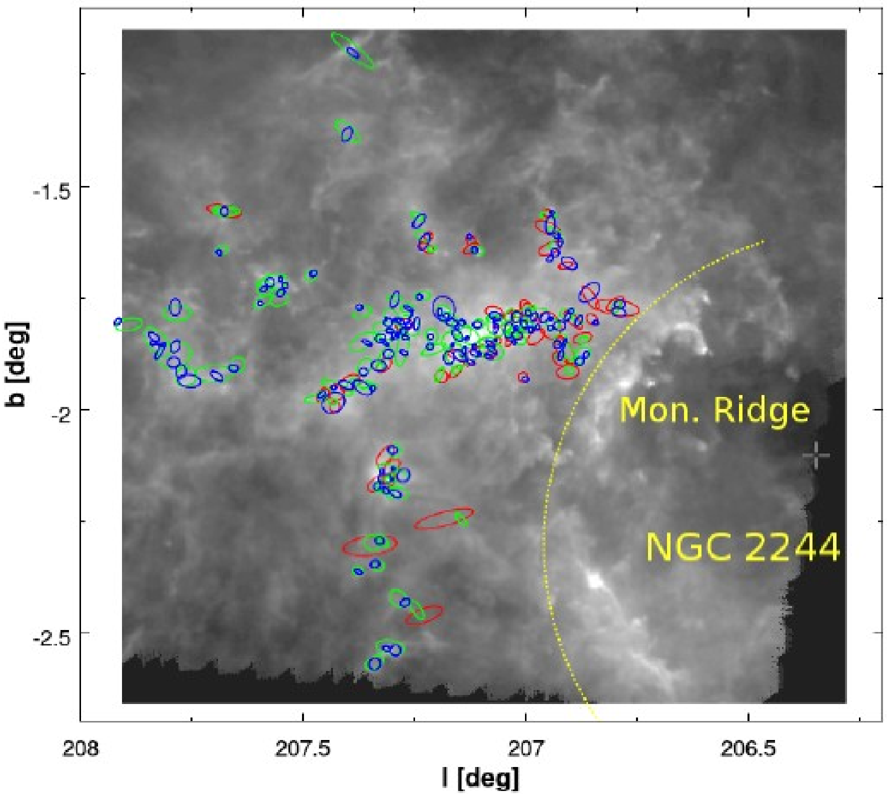


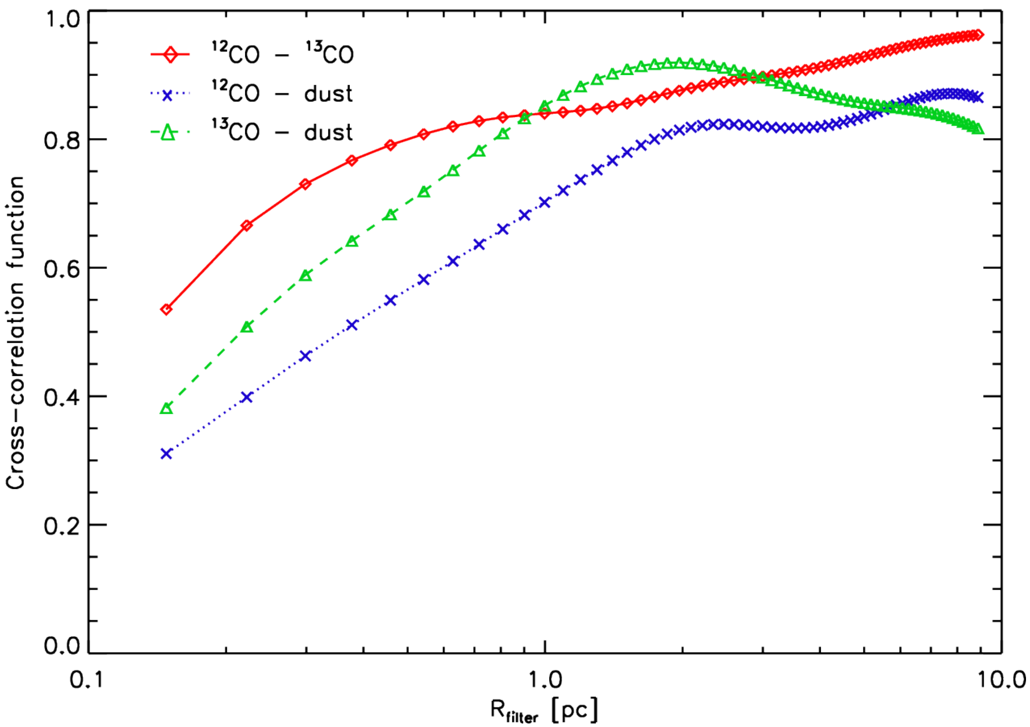
WWCC applications

Herschel and FCRAO maps of Rosette molecular cloud:

Analysis:

Comparison with dust map (Veltchev et al. 2017)



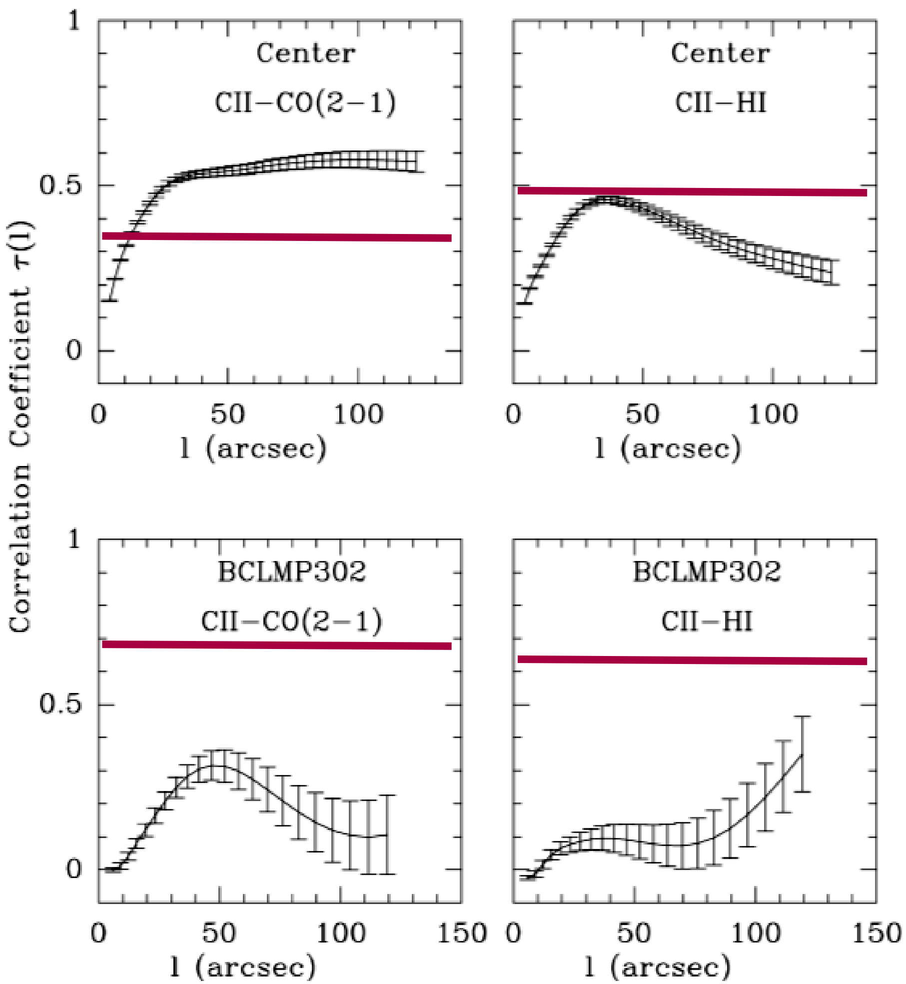


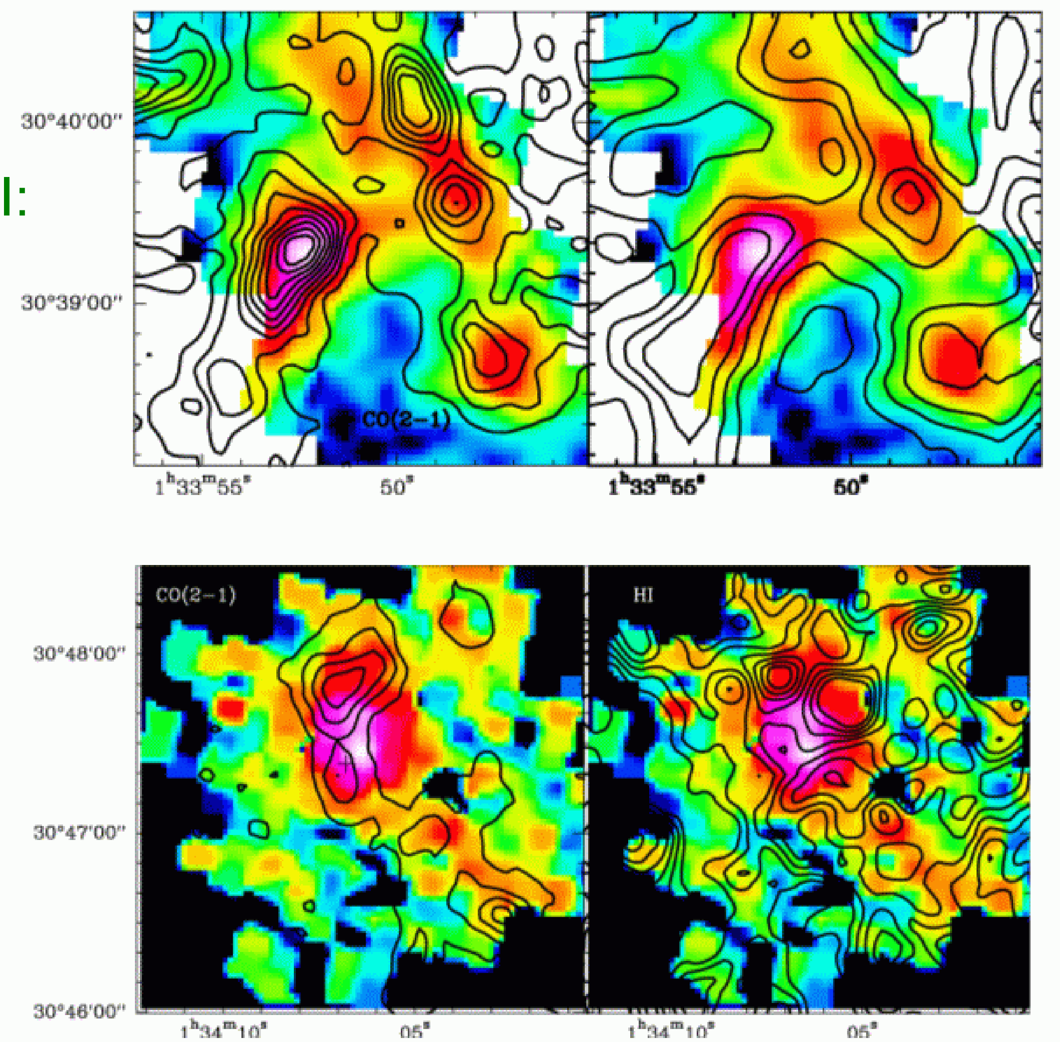
- ^{12}CO and ^{13}CO best correlate at small scales
- ^{13}CO good column density tracer at intermediate scales
- Negative trend at large scales

Conditions for CO dissociation unclear

Application to M33 maps

Correlation between [CII], CO 2-1, and HI:





Center(top) and BCLMP302 (right) maps,
Colours: [CII], contours: CO 2-1 (left), HI (right)
Mookerjea et al. (2015)

- Mismatch between global correlation and scale-dependent correlation
- Obvious anti-correlation of small-scale structures in maps
- Good global correlation does not prove origin from similar regions