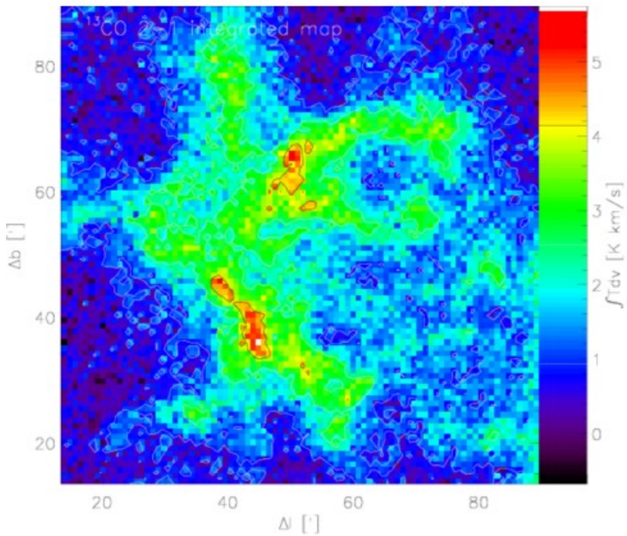


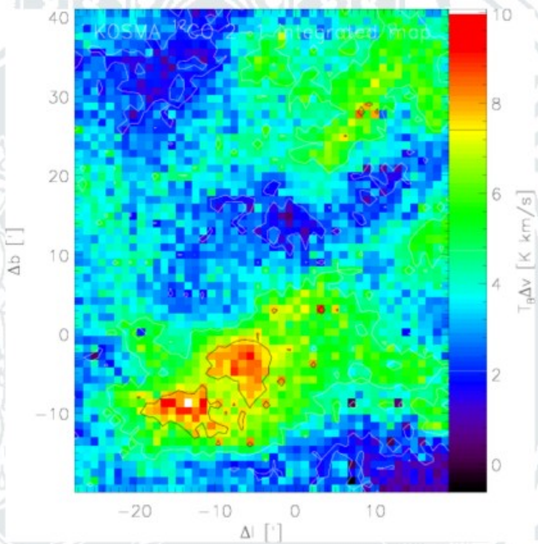


# Wavelet approaches for measuring interstellar cloud structure


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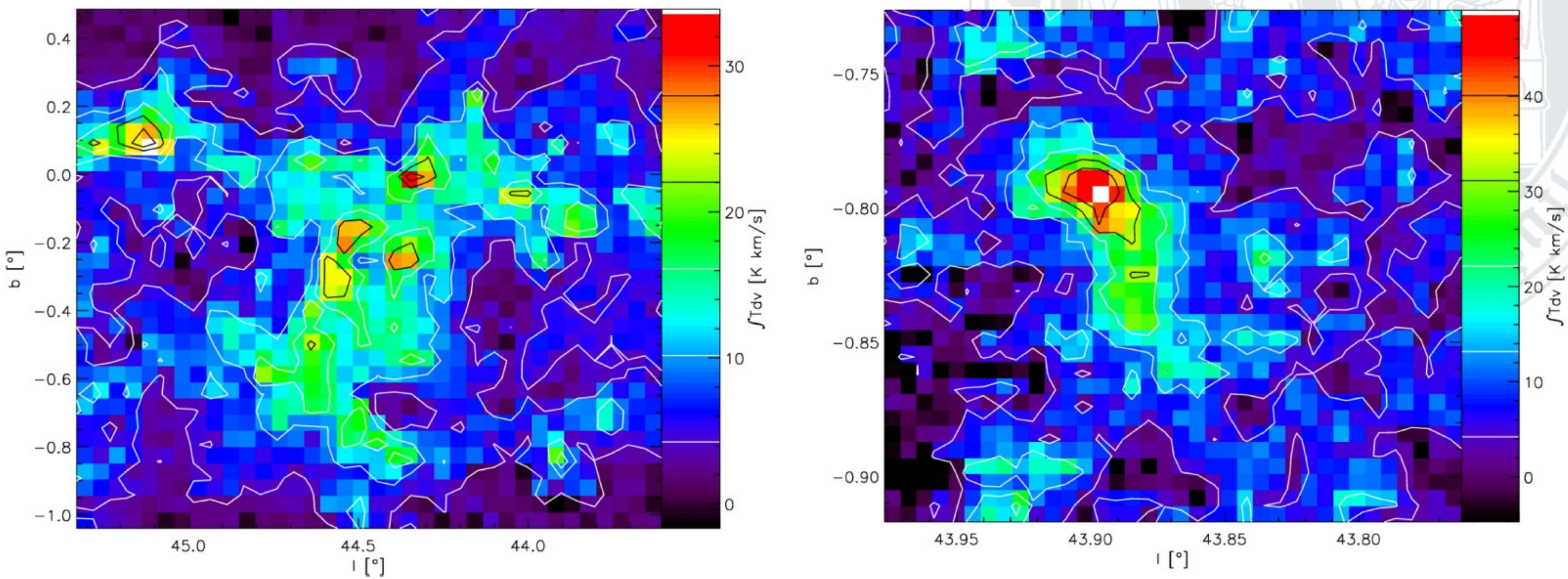
## Self similarity

- **Same type of structures on all scales**
  - Confirmed by observations:

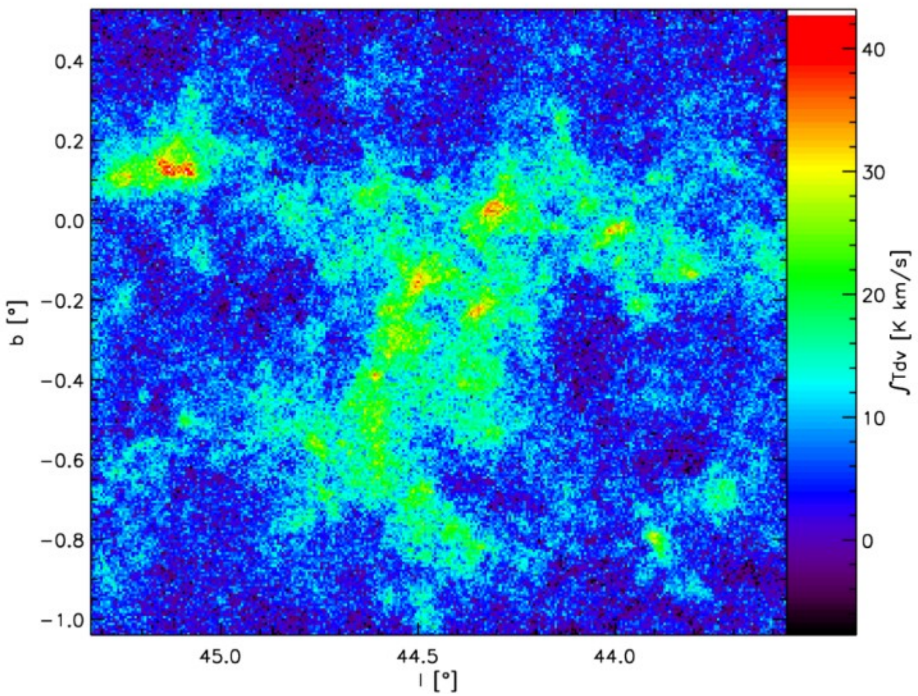


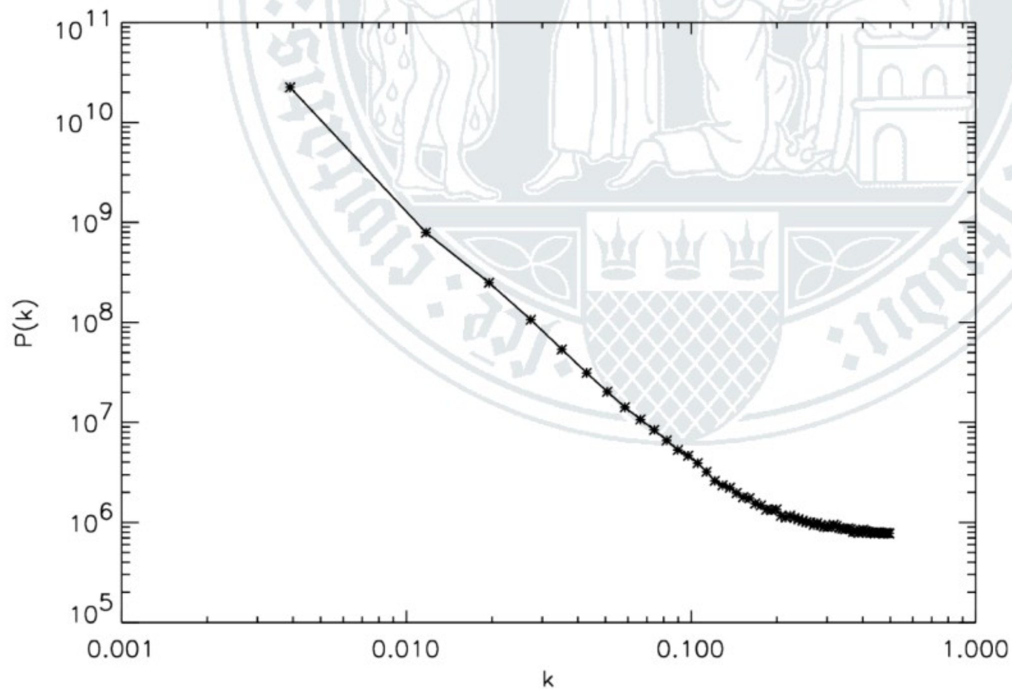

  - Integrated $^{13}CO$ 1-0 line map from the BU-FCRAO survey of the Galactic Ring at different “zoom levels” (Simon et al. 2002)
  - ☞ Self-similar over a large range of scales.

# Measure spatial scaling

## Power spectrum

Scale invariance results in a power-law power spectrum: $P(k) \propto k^{-\beta}$



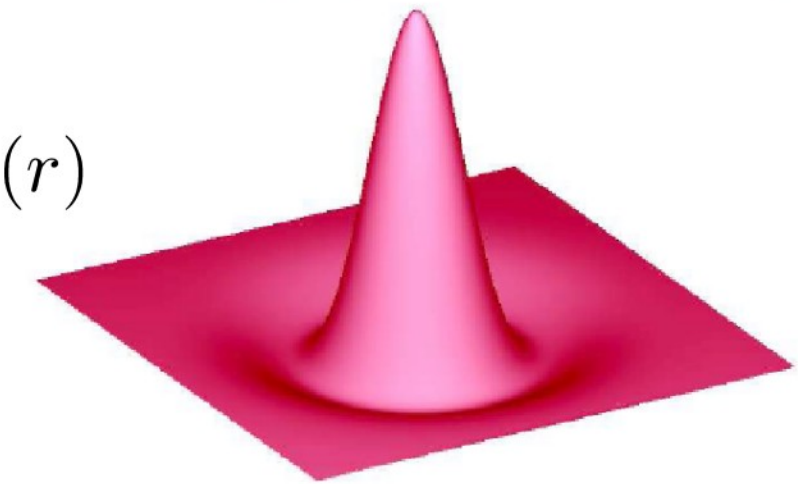

Analysis of G44.5 subfield of $^{13}$CO 1-0 line map from BU-FCRAO-GRS

- For rectangular maps, the power spectrum can be easily computed by FFT.

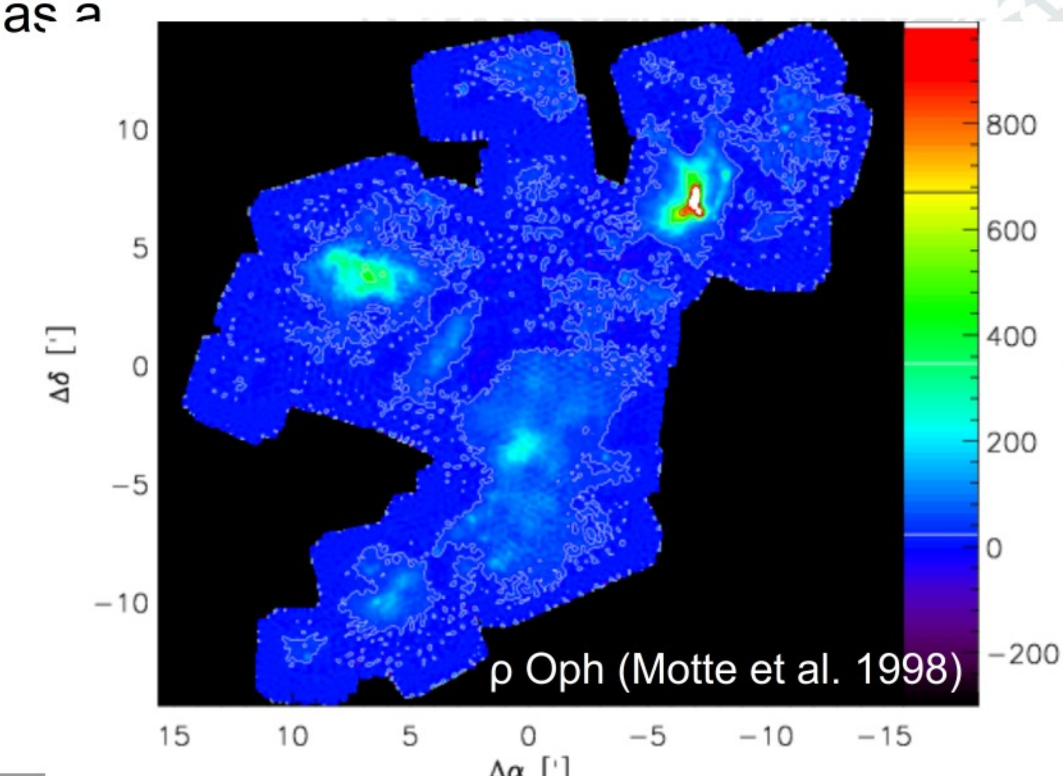
# Measure spatial scaling

## Δ-variance

**Filter map by radially symmetric wavelet** $\psi_l(r)$



- characteristic length scale *l*
- Measure variance in convolved image as function of the filter size *l*
- Gives relative amount of structure as a function of structure size

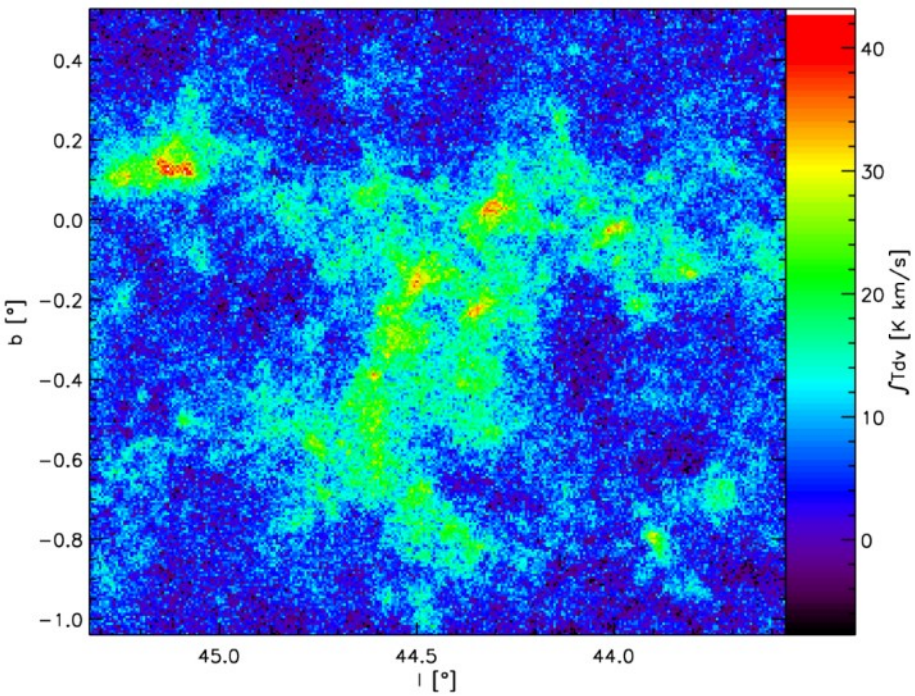


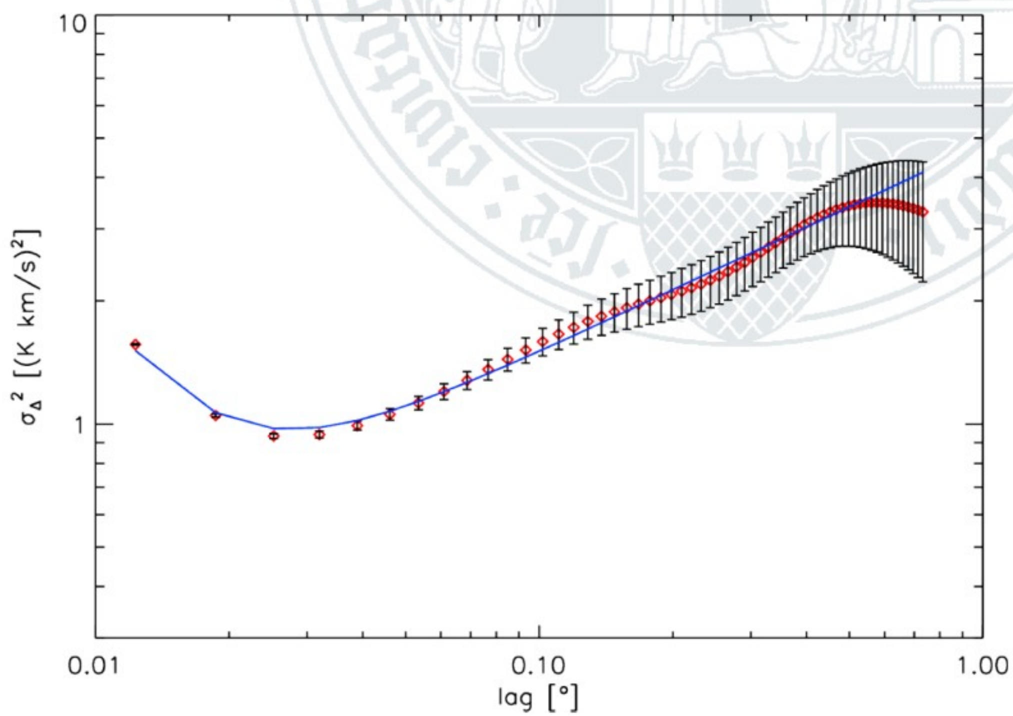

- **Advantages compared to power spectrum**
  - Δ-variance can measure spatial scaling of irregular maps, maps with variable noise.
  - In the application to observed maps the Δ-variance is much more robust.

## Δ-variance

Power-law power spectrum gives power-law Δ-variance: $\sigma_\Delta^2(l) \propto l^\alpha$

- Spectral index related to the power spectral index by $\alpha = \beta - 2$ .

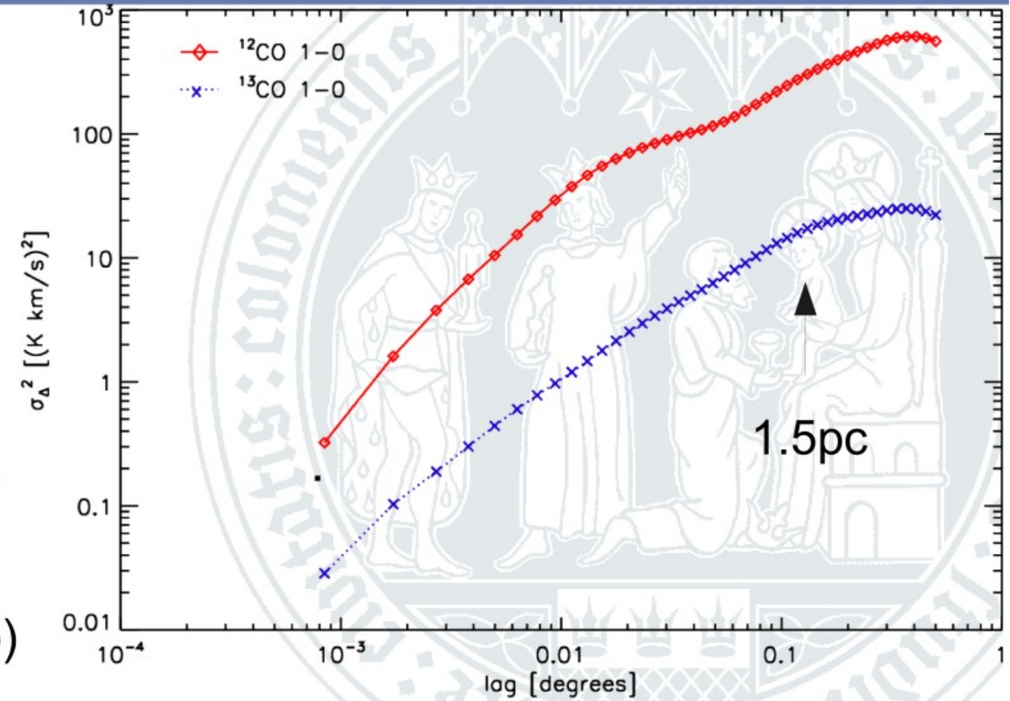


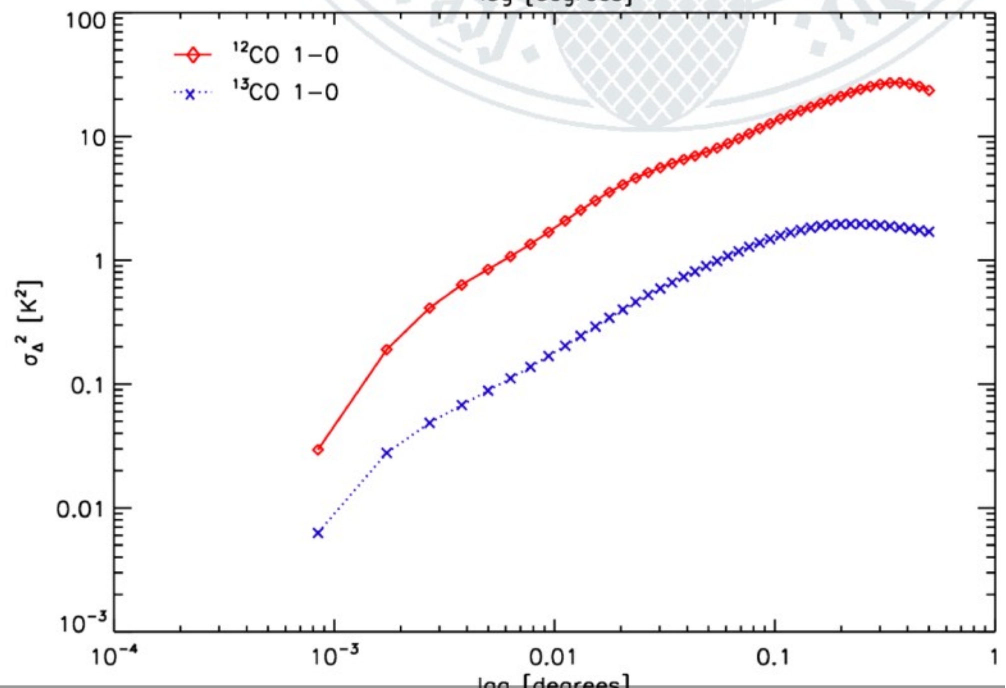

Analysis of G44.5 subfield of $^{13}CO$ 1-0 line map from BU-FCRAO-GRS

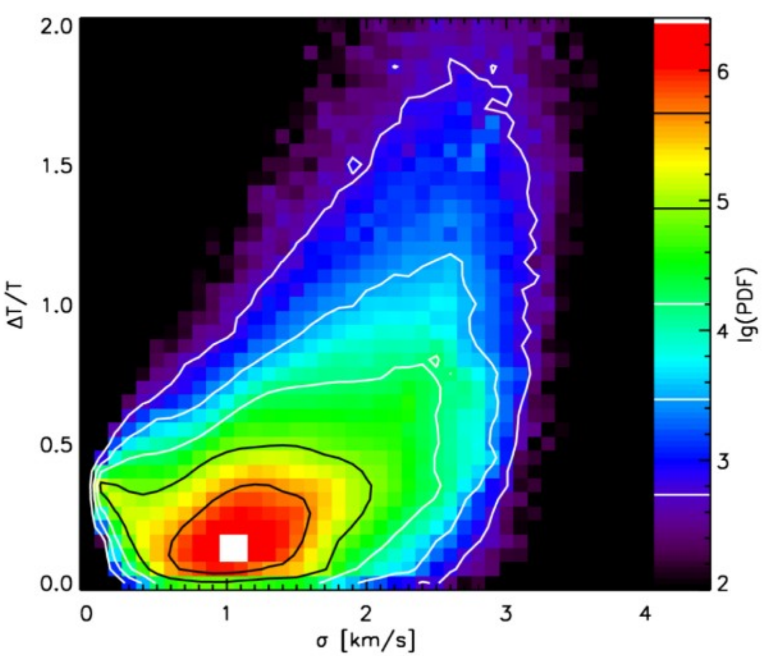
## Δ-variance spectra

- No artifacts from combination of single dish and interferometer or mosaicing
- Perfect power law for $^{13}CO$, β=3.1
- Some characteristic scales in all spectra




Δ-variance spectra of integrated maps (top) and 8.8km/s channel maps (bottom, Acre et al. in prep)



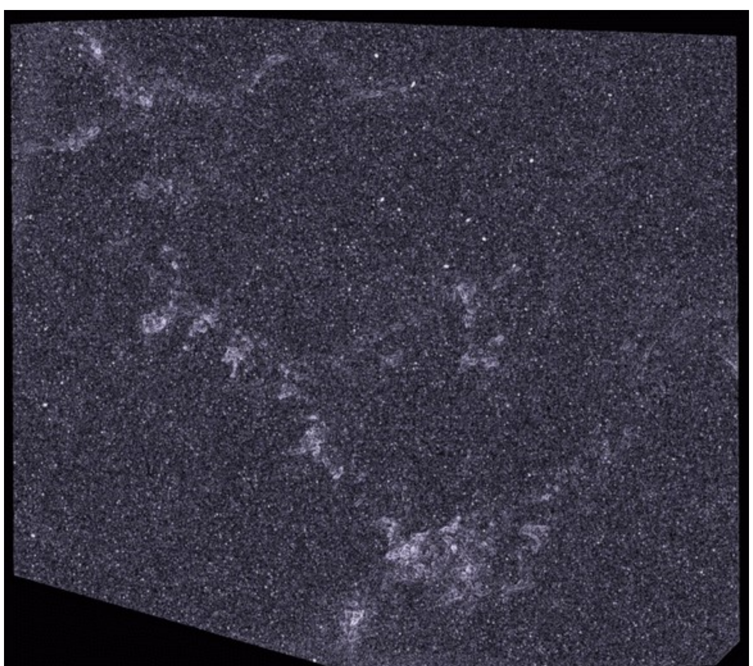

"Tauber"-line width plot for $^{13}CO$

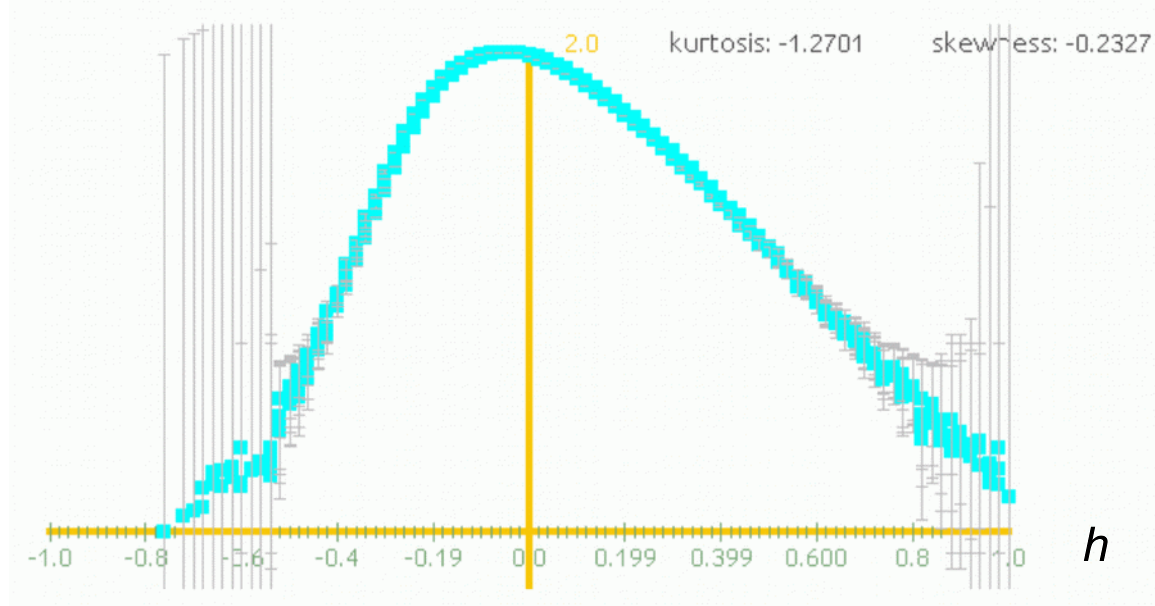
# Use for singularity spectrum

## INRIA approach (Hussein Yahia)

### Spatial distribution of box-car dimension at small scales

- *h(x)* is scaling exponent for 3-D “ball” around point x with radius r
- For $r \rightarrow 0$ (smallest resolution available), scales as $r^{h(x)} \rightarrow$ *h(x)* is the singularity exponent at *x*.
- Provide precise evaluation of the strength of fronts observables in clouds, and the possibility of computing fine statistics on them .



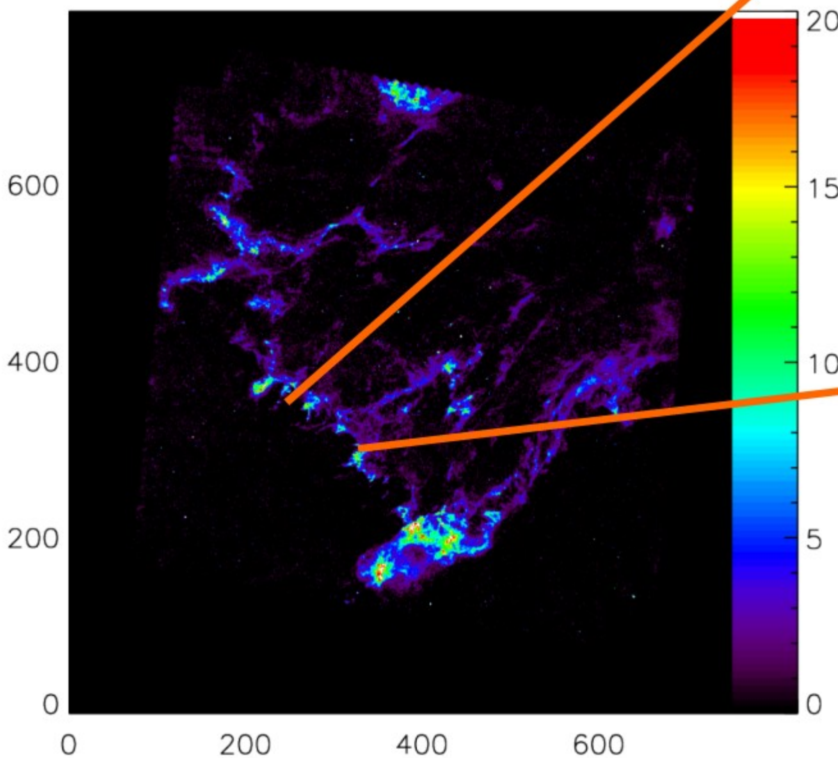


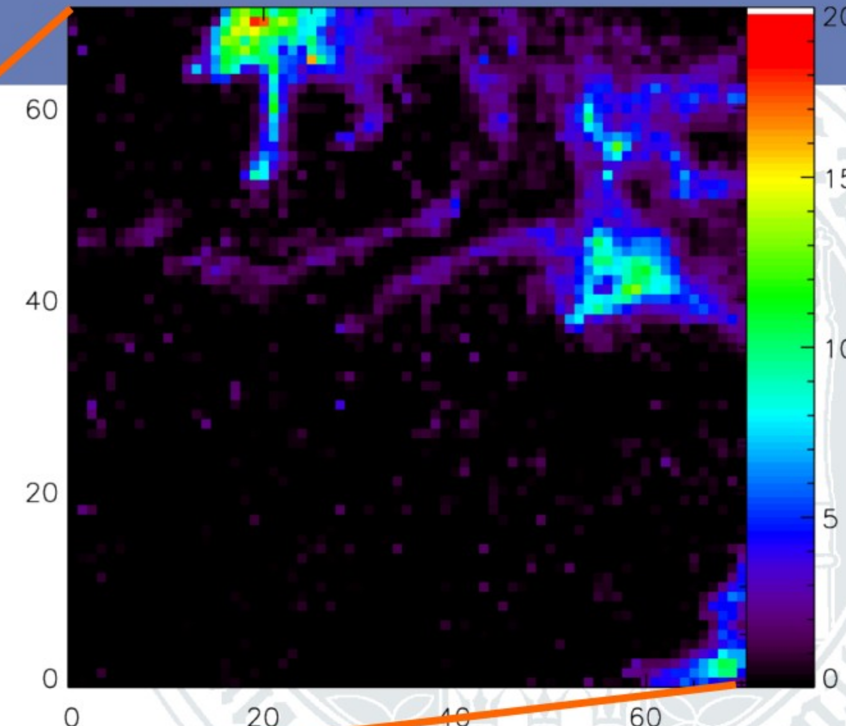

Singularity distribution for Draco 250 μm map

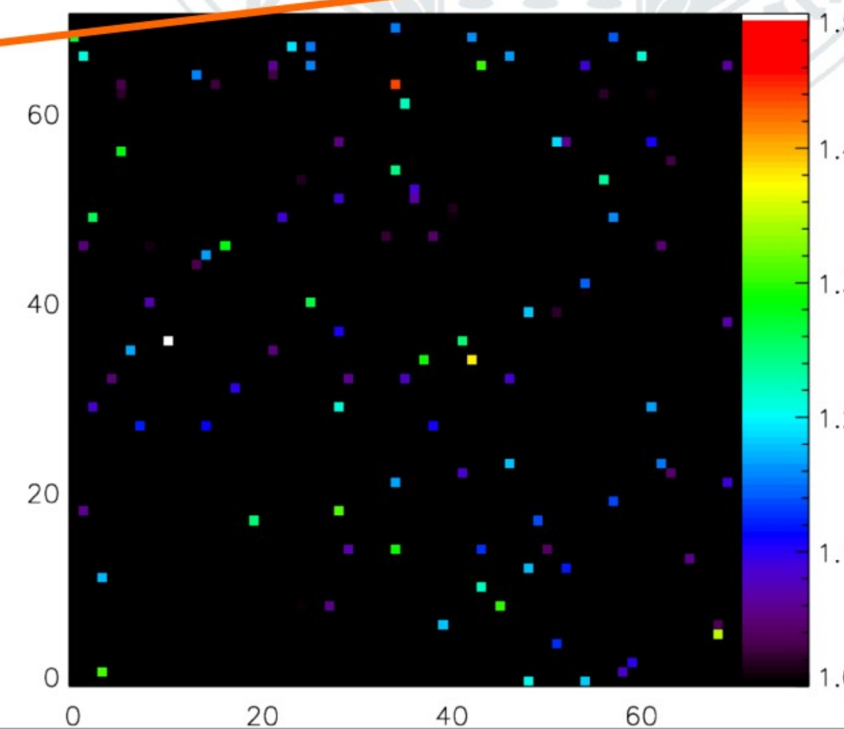
# Use for singularity spectrum

## Use Δ-variance slope at small scale instead of box-car dimension:

Application to same data set:




- Exponent threshold → Manifold $\mathcal{G}_0$

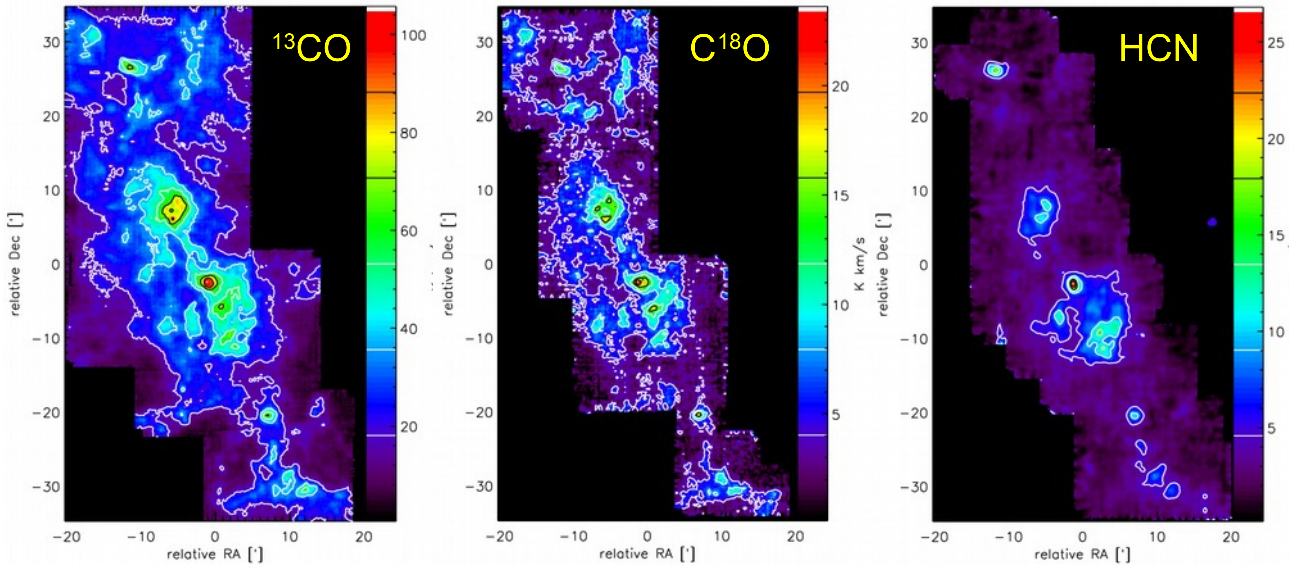
  → background galaxy detection

# Wavelet-weighted cross-correlation analysis (WWCC)

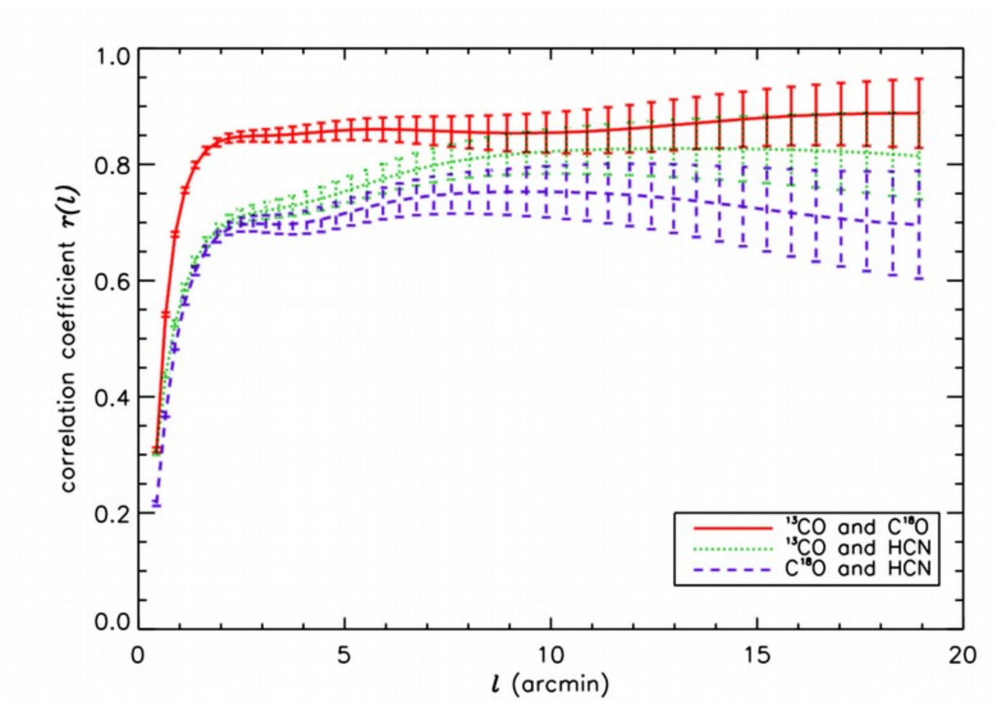
**WWCC:**

- Cross correlation in maps filtered by Δ-variance wavelet
- compare map structures depending on their scale

Apply to line observations of G333 (MOPRA):




(Arshakian & Ossenkopf 2015)




Result:

- $^{13}CO$-$C^{18}O$ perfectly correlated above the noise scale
- HCN behaves different at scales <7´ (~8pc)

**Difference in chemical structure or excitation conditions?**

# Wavelet-weighted cross-correlation analysis (WWCC)

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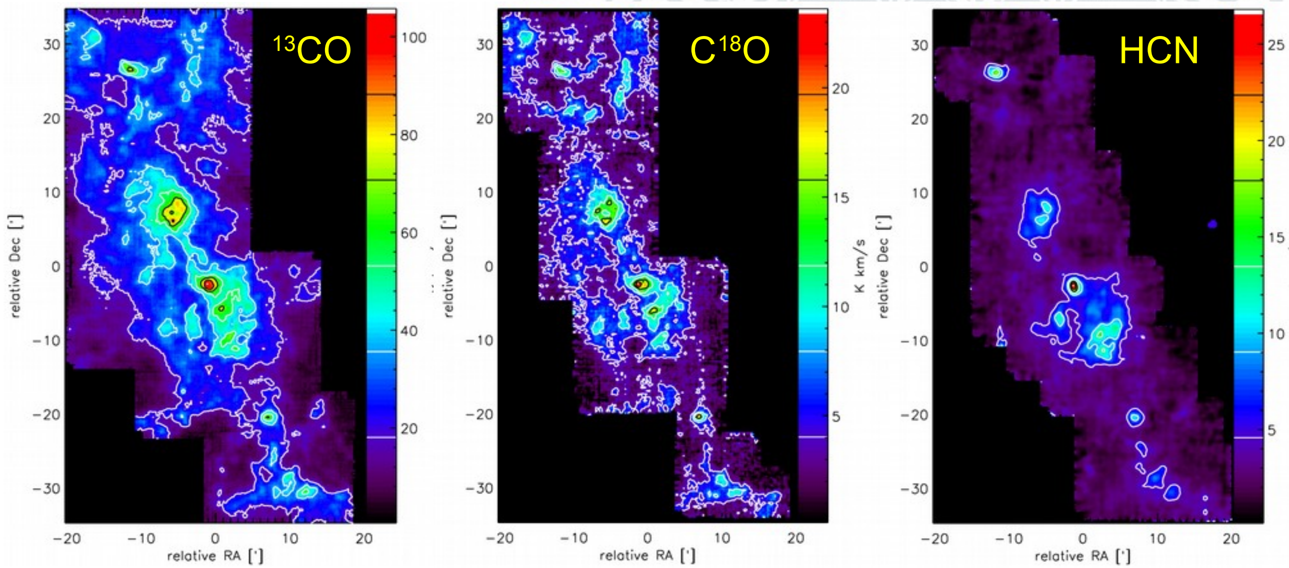
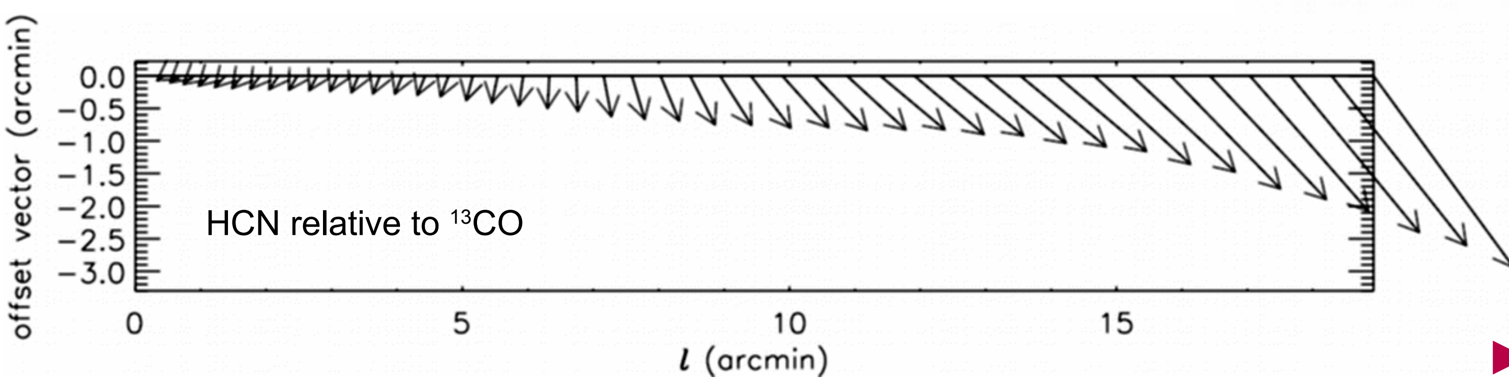



(Arshakian & Ossenkopf 2015)

*Scale-dependent offsets:*




- Increasing offset along the filament towards larger scales

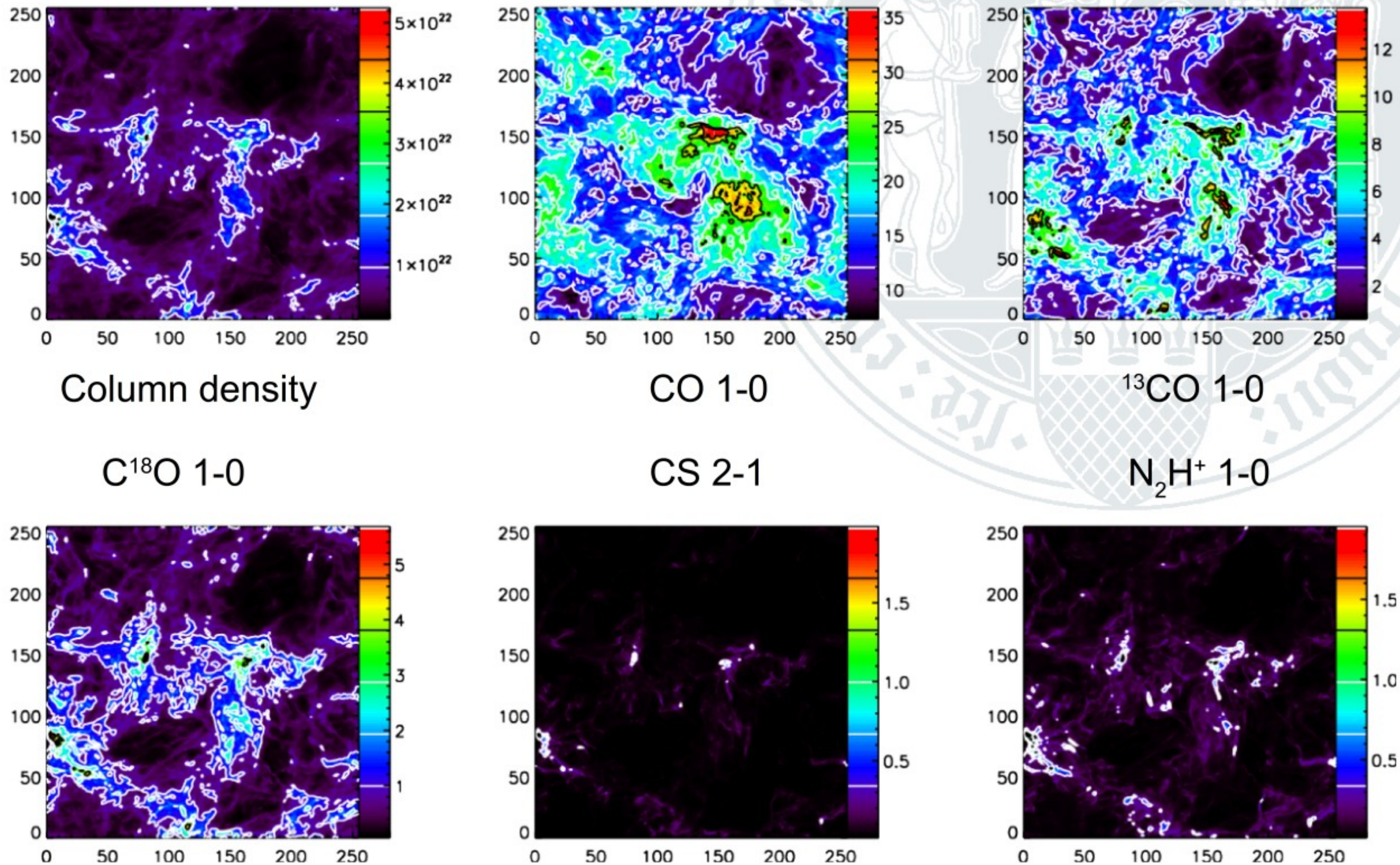
► cometary density structure or/and anisotropic radiation field

# Simulations: (M)HD models from Federrath & Klessen (2012)

## Compute line radiative transfer
(Simline3D, Ossenkopf 2002):

Integrated line intensities [K km/s]

Simplifications applied here:
- Constant abundances
- Isothermal




FK12 Model 20:

$M = 10\ M_\odot$

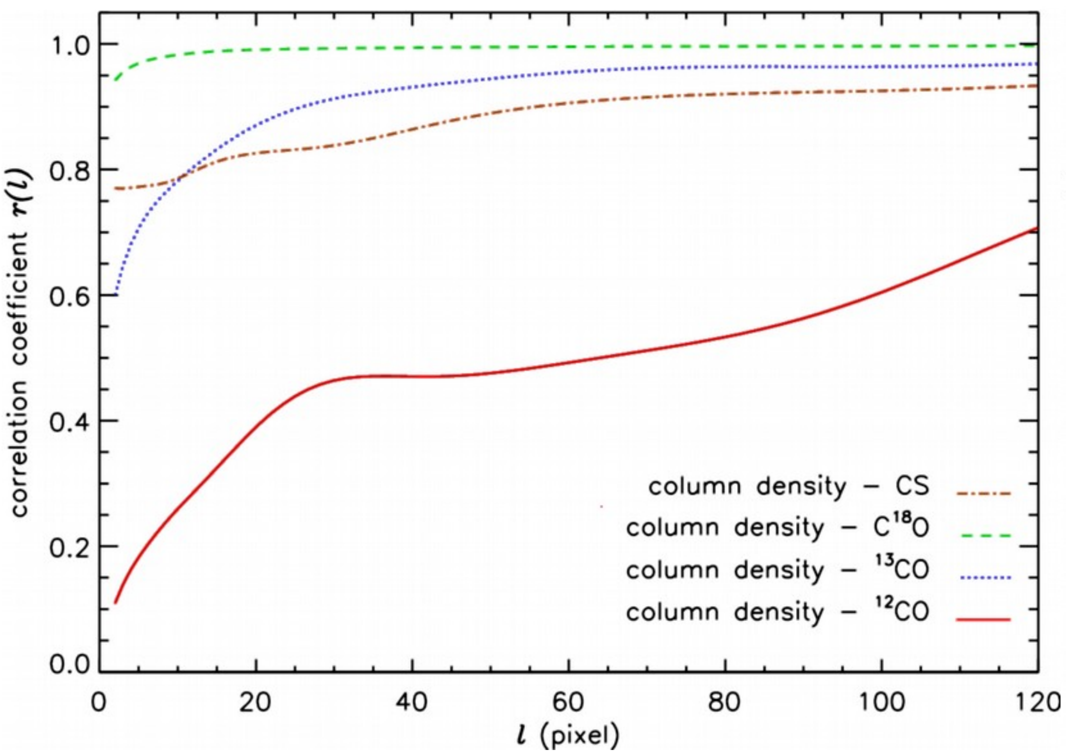
$D = 8\text{pc}$

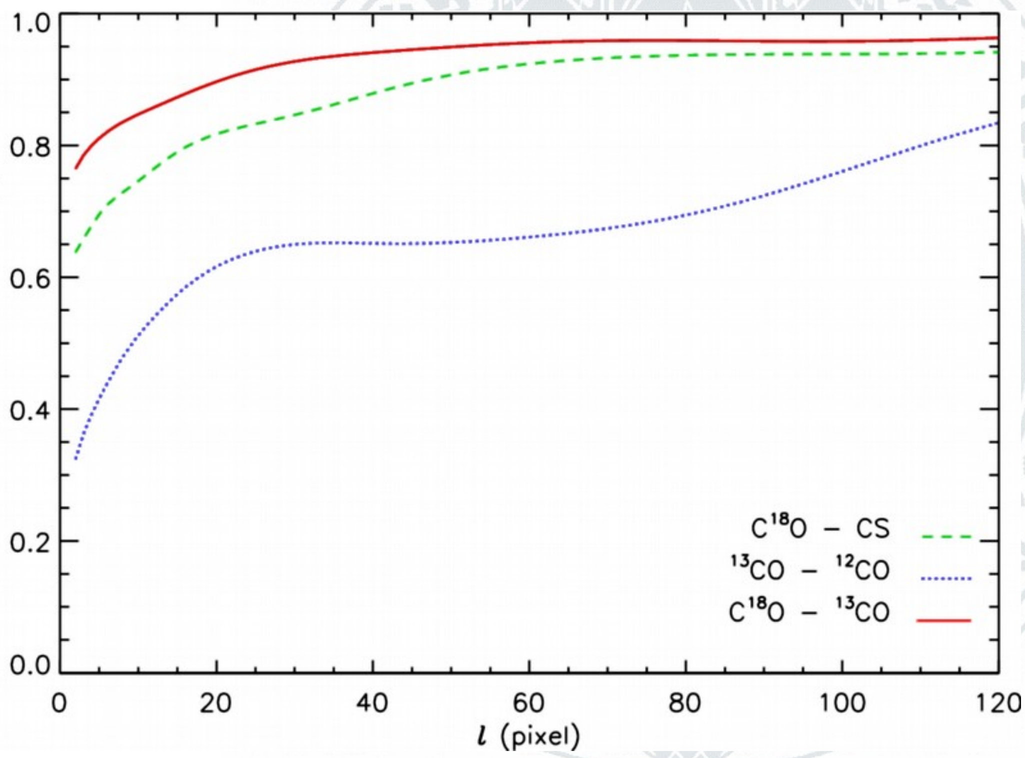
$<n> = 207\text{cm}^{-3}$

$<B> = 3\mu\text{G}$

- No line traces true column density,
- Rare CO isotopes best approximation

# Cross-correlation function




Results:

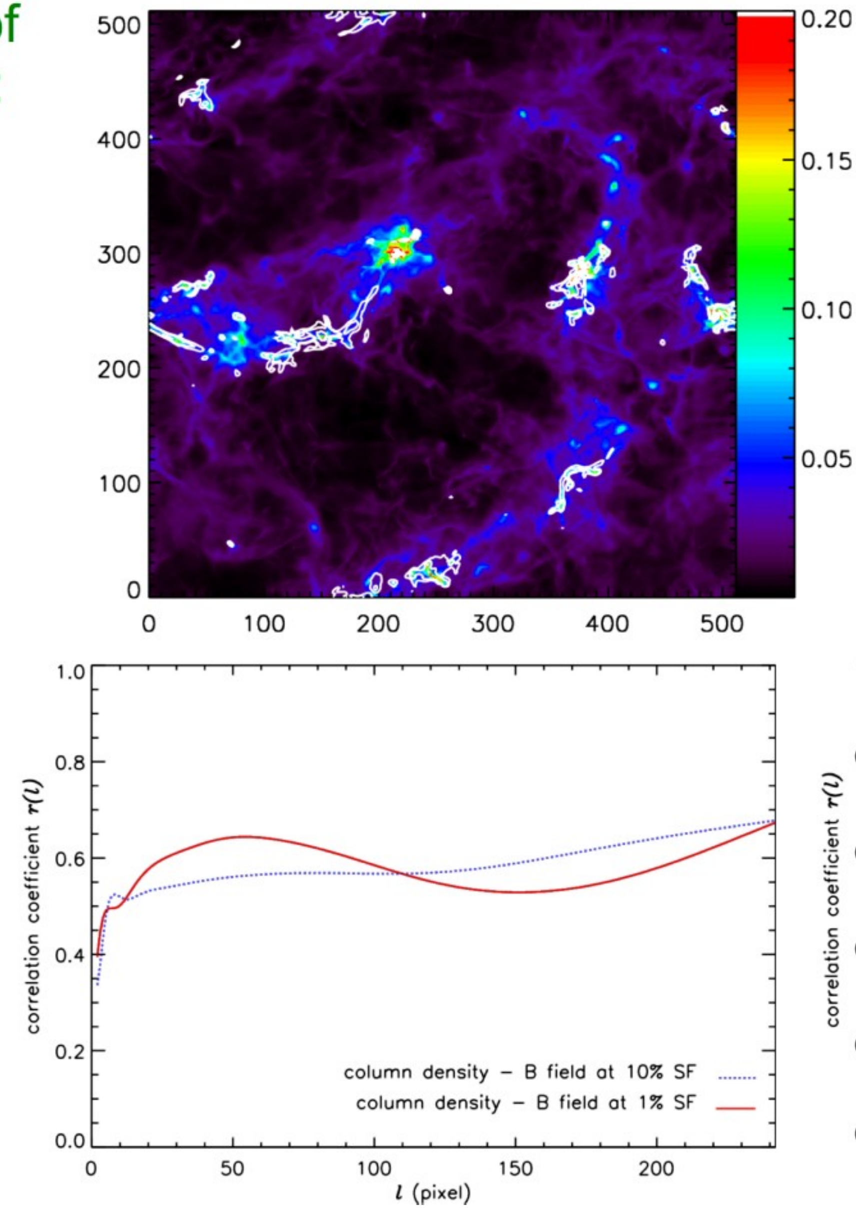
- Perfect correlation between spatial density structure and $C^{18}O$ emission
  - High correlation between $C^{18}O$ and $^{13}CO$ like in the observations
- Lower correlation for CS (like HCN in observations)
  - But no critical size scale
- $^{12}CO$ hardly reflects underlying density structure
  - only measures the velocity structure

# Cross-correlation function

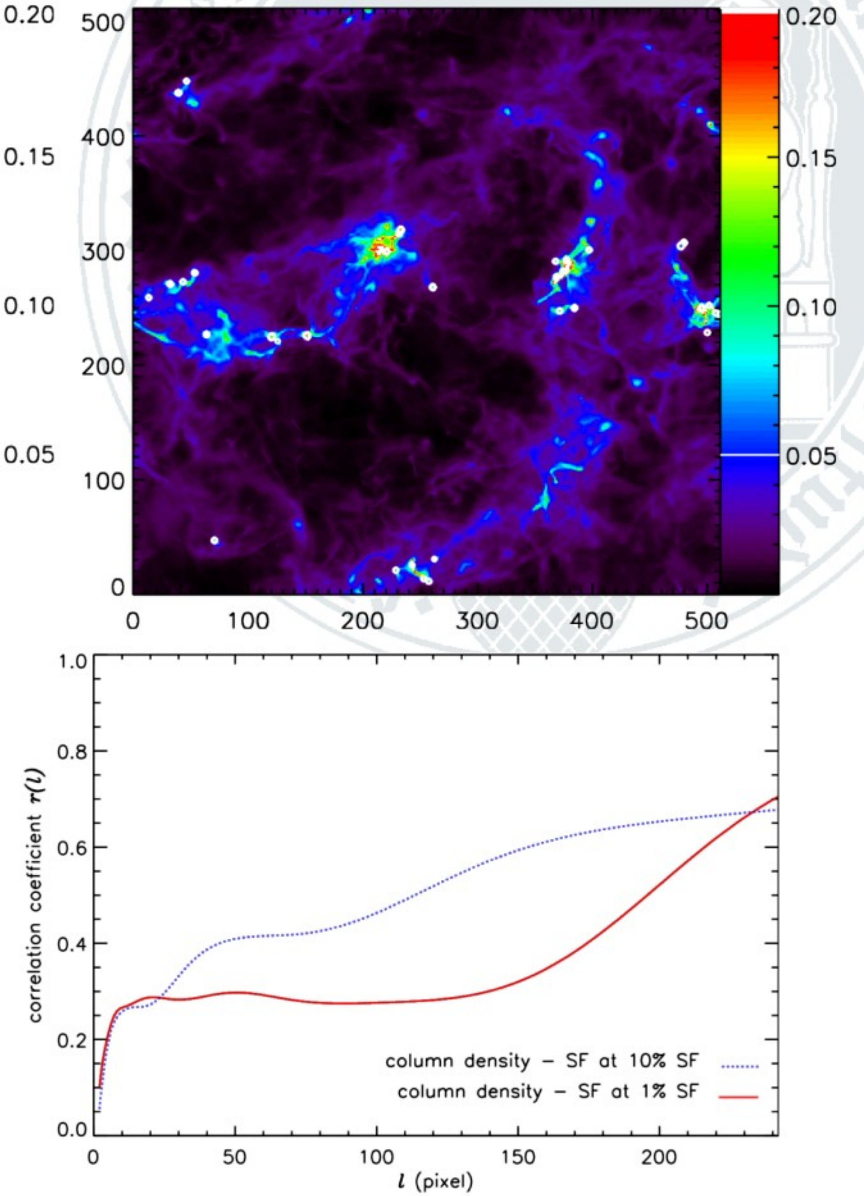
Compare direct properties of simulations:

- Weak and constant correlation between density and B-field
- Growing scale of SF feeding with time

Column density – magnetic field



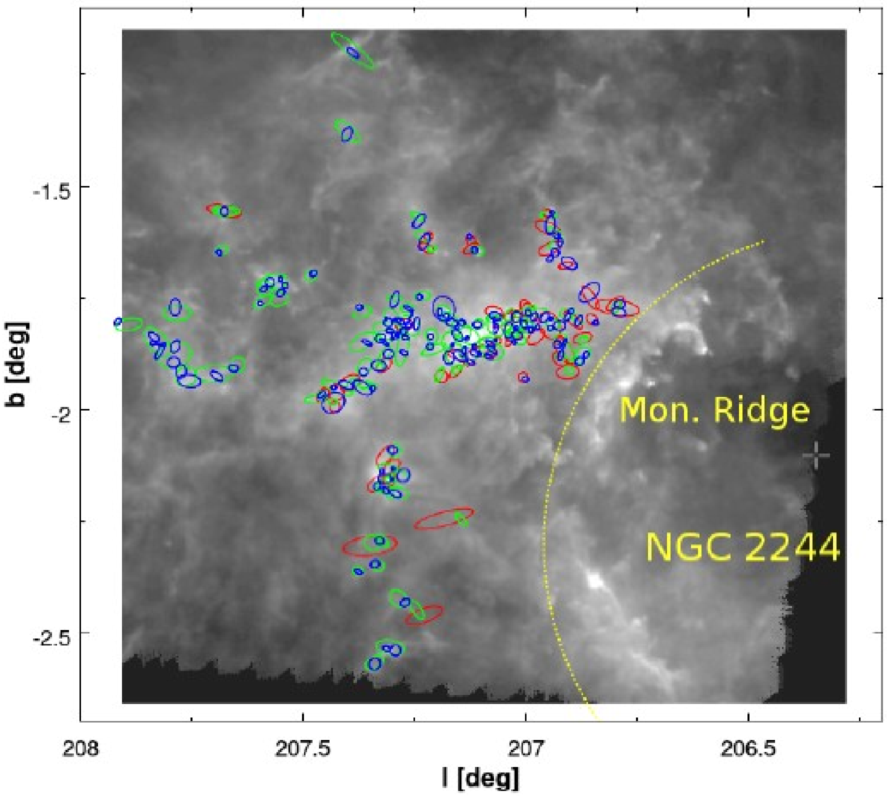

Column density – Protostellar cores

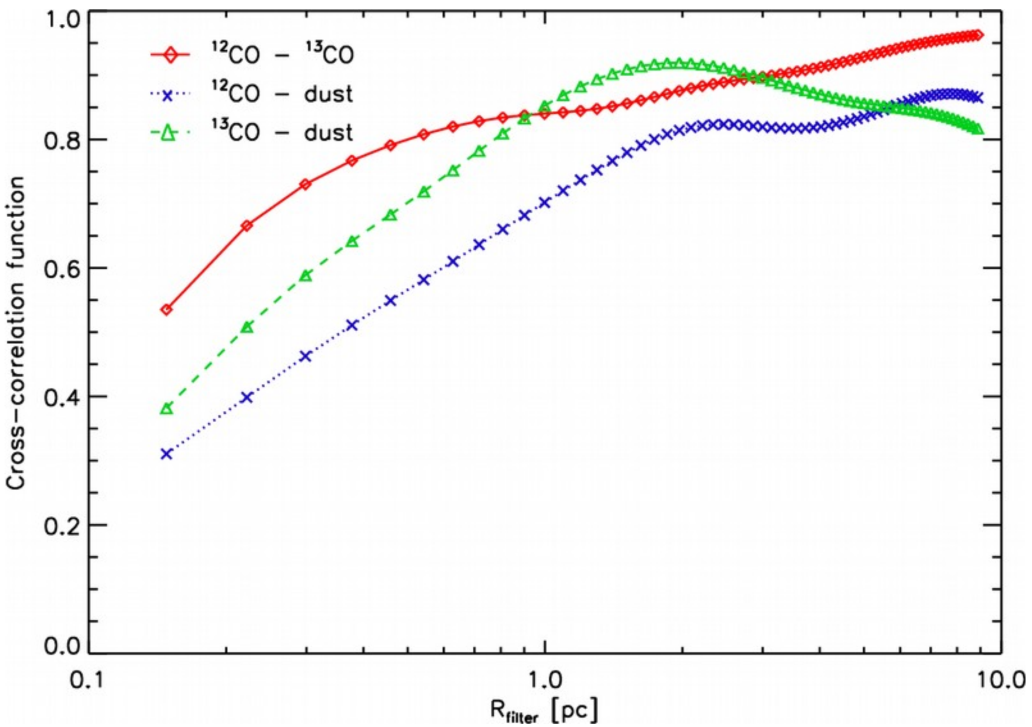


# WWCC applications

Herschel and FCRAO maps of
Rosette molecular cloud:

Analysis:
Comparison with dust map
(Veltchev et al. 2017)



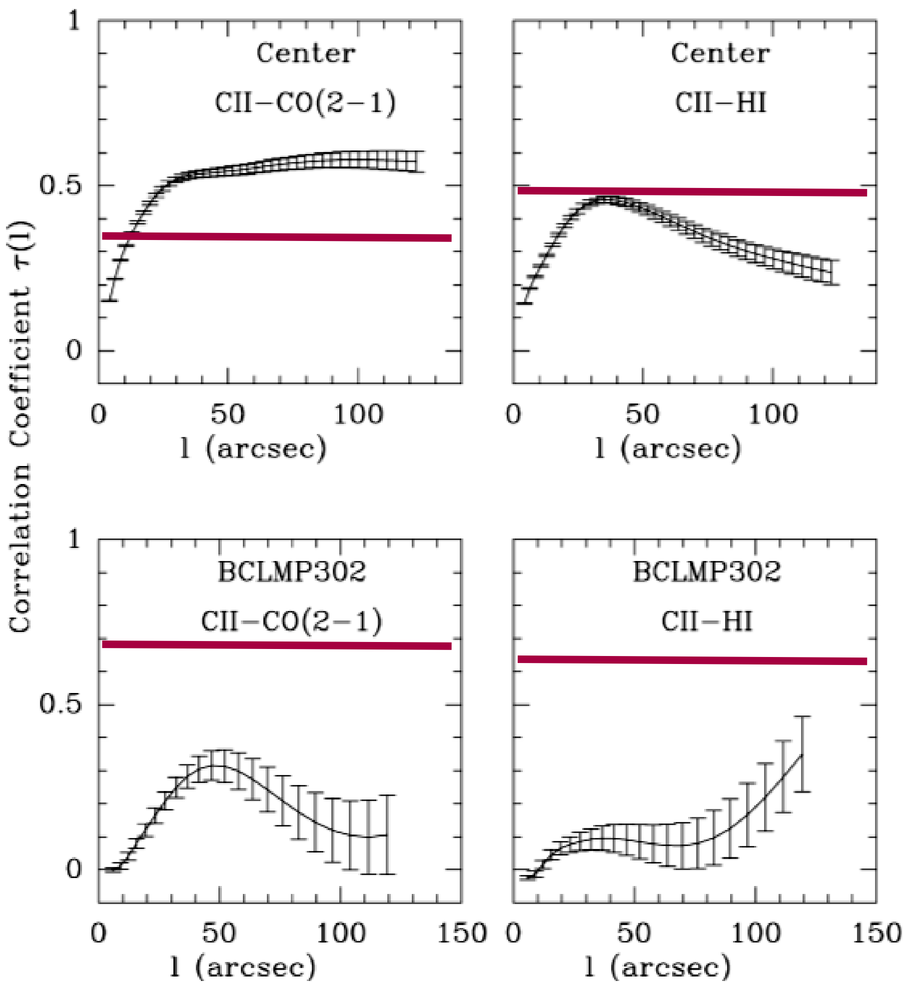



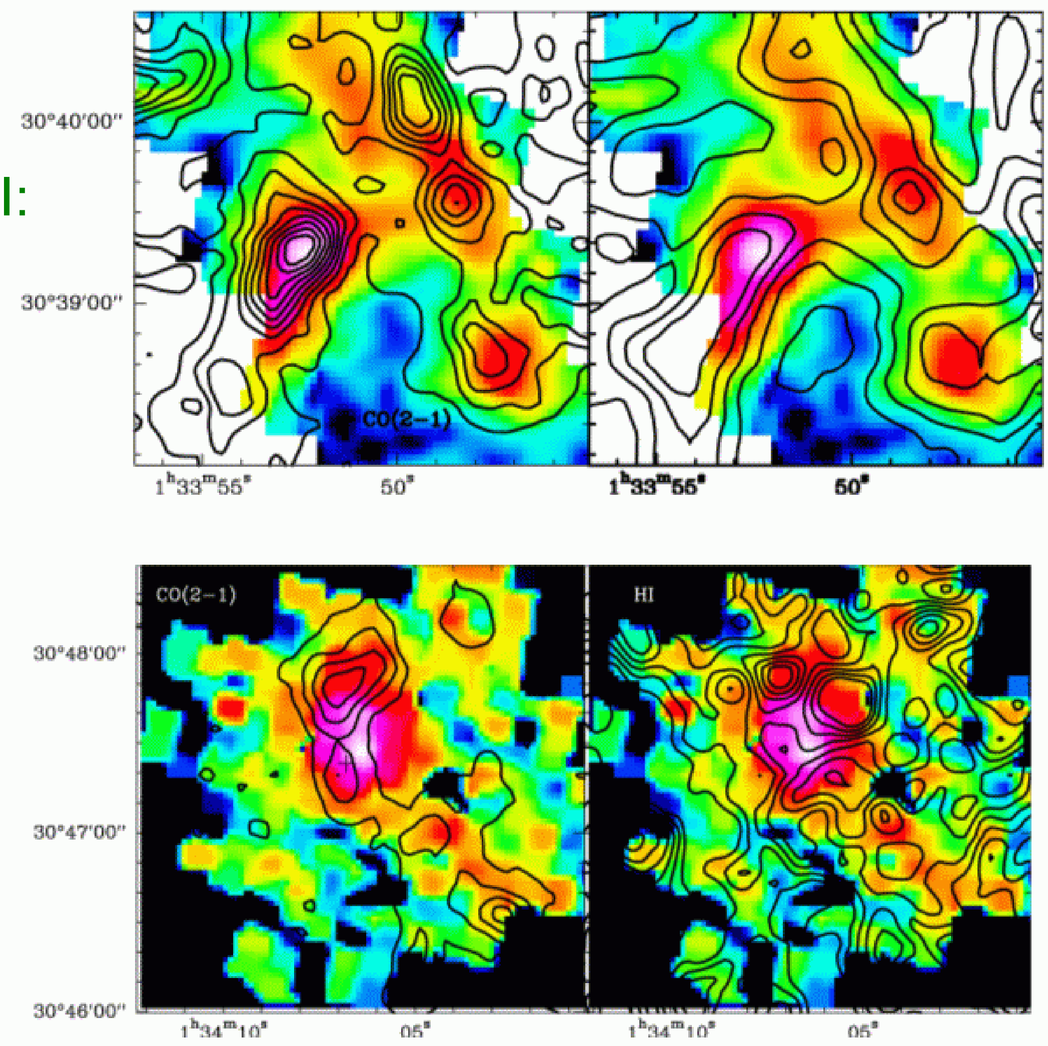

- $^{12}$CO and $^{13}$CO best correlate at small scales
- $^{13}$CO good column density tracer at intermediate scales
- Negative trend at large scales

**Conditions for CO dissociation unclear**

# Application to M33 maps

## Correlation between [CII], CO 2-1, and HI:




Center(top) and BCLMP302 (right) maps,
Colours: [CII], contours: CO 2-1 (left), HI (right)
Mookerjea et al. (2015)

- Mismatch between global correlation and scale-dependent correlation
- Obvious anti-correlation of small-scale structures in maps
- Good global correlation does not prove origin from similar regions