HIFI calibration: Basic concepts

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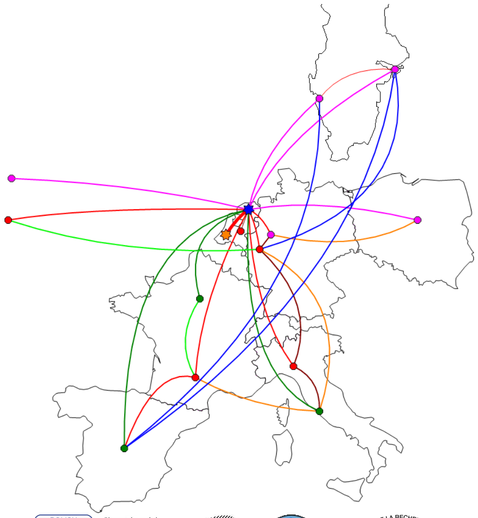

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Fundamental problem

- **4-dimensional problem**
  - 3-dimensional point-spread function:
    - Pointing calibration
    - Frequency calibration
    - Intensity calibration
      - Includes beam-calibration
- **System response is a strong function of frequency**
  - Cross-talk of different frequencies
    - HIFI is a double- (multi-) sideband instrument
    - No accurately known reference line sources available
- **Main problem in instrument calibration:**
  - Calibration of a line instrument through continuum sources = superposition of sidebands

Intensity calibration

- Determine J_S from:

$$
\begin{aligned}
c \;=\;& \gamma_{\mathrm{ssb}}\left\{\eta_{\mathrm{l,ssb}}\left[\eta_{\mathrm{sf,ssb}}J_{\mathrm{S,ssb}} + (1-\eta_{\mathrm{sf,ssb}})J_{\mathrm{R,ssb}}\right] + (1-\eta_{\mathrm{l,ssb}})J_{\mathrm{T,ssb}}\right\} \\
& + \gamma_{\mathrm{isb}}\left\{\eta_{\mathrm{l,isb}}\left[\eta_{\mathrm{sf,isb}}J_{\mathrm{S,isb}} + (1-\eta_{\mathrm{sf,ssb}})J_{\mathrm{R,isb}}\right] + (1-\eta_{\mathrm{l,isb}})J_{\mathrm{T,isb}}\right\} \\
& + \gamma_{\mathrm{rec}}J_{\mathrm{rec}} + z
\end{aligned}
$$

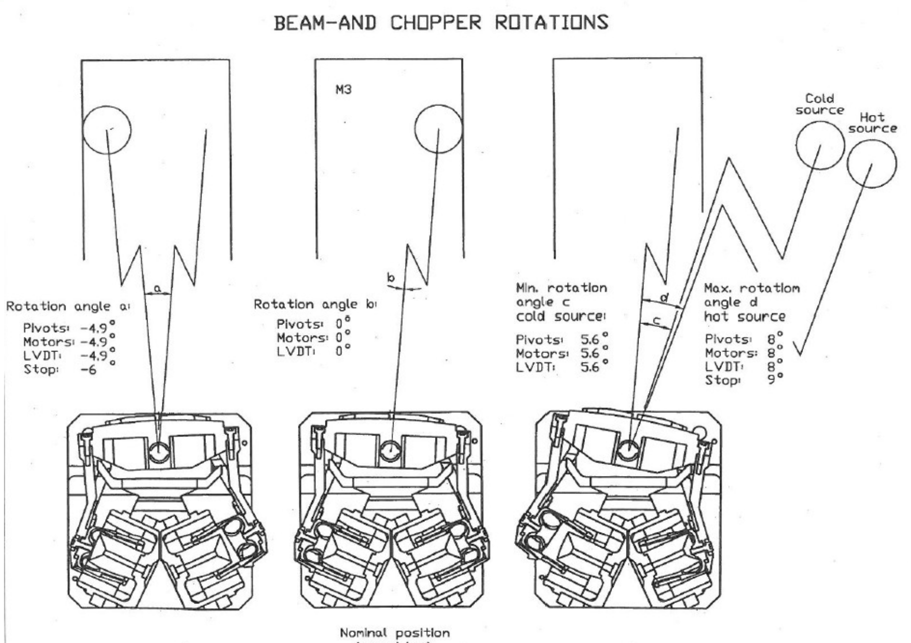
  - bandpass/gain functions γ and efficiencies η may differ for each spectrometer channel

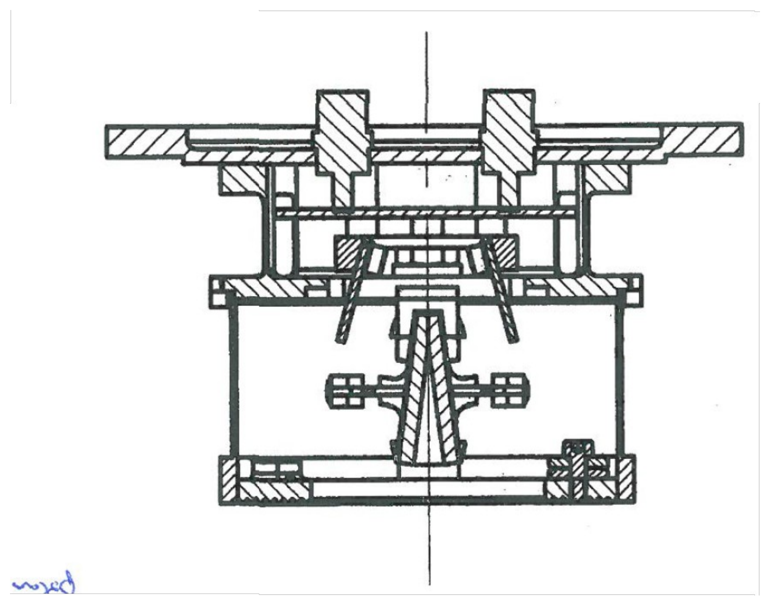
- Simplifications used:
  - Assuming linear response and linear superposition
  - Split between bandpass γ which is rapidly varying with channel number and slowly varying efficiencies
  - Neglect sideband imbalances for coupling factors and receiver noise

Intensity calibration

3-point calibration:

- Cold load, hot load, blank sky
- Allows to determine bandpass γ_{rec}, receiver noise J_{rec} and forward efficiency η_{l} simultaneously
- The analysis of the spectral ripple in the blank sky measurement (OFF calibration) allows to fit the standing waves, w_{ssb} and w_{isb}, using a model.





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Bandpass calibration

- Two-load chopper wheel calibration to determine intensity bandpass $\gamma_{\rm rec}$ and receiver temperature $J_{\rm rec}$.
- Both quantities are computed fully channel dependent.

Receiver parameters:

$$
\begin{aligned}
\gamma^{\rm l}_{\rm rec} &= \frac{c_{\rm hot} - c_{\rm cold}}{(\eta_{\rm h} + \eta_{\rm c} - 1)(J_{\rm h,eff} - J_{\rm c,eff})} \\
J^{\rm l}_{\rm rec} &= \frac{\eta_{\rm h}(c_{\rm cold} - z) - (1 - \eta_{\rm c})(c_{\rm hot} - z)}{c_{\rm hot} - c_{\rm cold}}(J_{\rm h,eff} - J_{\rm c,eff}) - J_{\rm c,eff} \\
&= \frac{(\eta_{\rm h} + Y\eta_{\rm c} - Y)J_{\rm h,eff} - (\eta_{\rm h} + Y\eta_{\rm c} - 1)J_{\rm c,eff}}{Y - 1}
\end{aligned}
$$

with

$$
\begin{aligned}
Y &= \frac{c_{\rm hot} - z}{c_{\rm cold} - z} \\
J_{\rm eff} &= G_{\rm ssb}J_{\rm ssb} + (1 - G_{\rm ssb})J_{\rm isb}
\end{aligned}
$$

→ Success: loads very stable!

OFF calibration

- Blank sky calibration measurement to determine telescope coupling η_l and standing wave pattern

The standing wave contribution to the radiation field can be extracted from the count rate of the OFF measurement c_{OFF}:

$$J_{\mathrm{sw}} = \frac{c_{\mathrm{OFF}} - z}{\gamma^{\mathrm{l}}_{\mathrm{rec}}} - J^{\mathrm{l}}_{\mathrm{rec}} - \eta_{\mathrm{l}} J_{\mathrm{R,eff}}$$

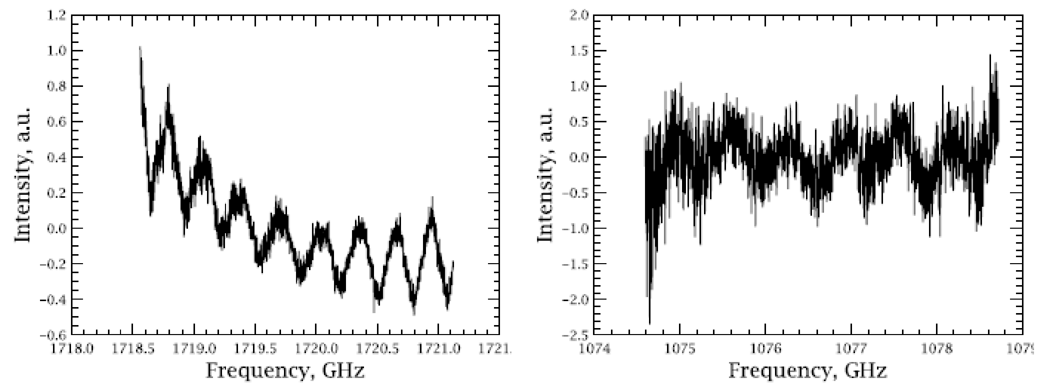

Fig. 5. Examples of the 320 MHz standing wave seen in the HEB IF chain(left panel) and the 680 MHz standing wave due to reflection between the mixer focus and the diplexer rooftop mirror (right panel)

The frequency dependent part of the blank-sky observation contains the standing waves

Standing waves

Not clear which standing mechanism dominates:

- **Additive standing waves:**

$$(1-\eta_l)J_{\mathrm{T,eff}} + w_{\mathrm{ssb}} + w_{\mathrm{isb}} = \frac{c_{\mathrm{OFF}} - z}{\gamma^{l}_{\mathrm{rec}}} - J^{l}_{\mathrm{rec}} - \eta_l^{\mathrm{guess}} J_{\mathrm{R,eff}}$$

- **Standing waves changing the coupling to the telescope:**

$$(1-\eta_l)\,J_{\mathrm{T,eff}} + (1 \pm b_{\mathrm{T}}\nu_{\mathrm{IF}})\,(W_{\mathrm{ssb}} + W_{\mathrm{isb}})\,J_{\mathrm{T,LO}} = \frac{c_{\mathrm{OFF}} - z}{\gamma^{l}_{\mathrm{rec}}} - J^{l}_{\mathrm{rec}} - \eta_l^{\mathrm{guess}} J_{\mathrm{R,eff}}$$

$$W_{\mathrm{ssb}} = w_{\mathrm{ssb}} G_{\mathrm{ssb}} \qquad W_{\mathrm{isb}} = w_{\mathrm{isb}}(1 - G_{\mathrm{ssb}})$$

- **Standing waves changing overall gain:**

$$W_{\mathrm{ssb}} = w_{\mathrm{ssb}} \frac{(1-\eta_l)}{\gamma^{l}_{\mathrm{rec}}}$$

$$W_{\mathrm{isb}} = w_{\mathrm{isb}} \frac{(1-\eta_l)}{\gamma^{l}_{\mathrm{rec}}}$$

- The standing waves from both sidebands are always superimposed
- Model needed to disentangle

OFF calibration

- Blank sky measurement to get telescope coupling η_l and standing waves
- Assumptions:
  - Standing waves modulate contributions from
    - astronomical source
    - telescope
    - receiver
  - Standing waves are stable
- Reality (see David's talk):
  - Standing waves to telescope negligible
  - Gain standing waves (IF path, LO standing waves) unstable
    - covered by AOT design
  - SW to loads, i.e. frequency-dependence of η_h, η_c much larger
  - OFF measurements never fully analysed
  - Model for sideband separation in standing waves still missing

Intensity calibration

Resulting calibration equations:

We can distinguish two basic source calibration schemes:

Modes without baseline calibration measurement (e.g. position switch):

$$J_{\mathrm{source}} - J_{\mathrm{OFF}} = \frac{1}{G_{\mathrm{ssb}}\eta_{\mathrm{l}} + w_{\mathrm{ssb}}} \frac{\eta_{\mathrm{h}} + \eta_{\mathrm{c}} - 1}{\eta_{\mathrm{sf}}} \times \frac{(c_{\mathrm{source}} - c_{\mathrm{OFF}})}{c_{\mathrm{hot}} - c_{\mathrm{cold}}} (J_{\mathrm{h,eff}} - J_{\mathrm{c,eff}})$$

Modes with baseline calibration measurement (e.g. frequency-switch):

$$J_{\mathrm{source}} - J_{\mathrm{OFF}} = \frac{1}{G_{\mathrm{ssb}}\eta_{\mathrm{l}} + w_{\mathrm{ssb}}} \frac{\eta_{\mathrm{h}} + \eta_{\mathrm{c}} - 1}{\eta_{\mathrm{sf}}} \times \frac{(c_{\mathrm{S}}^{\mathrm{source}} - c_{\mathrm{R}}^{\mathrm{source}}) - (c_{\mathrm{S}}^{\mathrm{OFF}} - c_{\mathrm{R}}^{\mathrm{OFF}})}{c_{\mathrm{hot}} - c_{\mathrm{cold}}} (J_{\mathrm{h,eff}} - J_{\mathrm{c,eff}})$$

The actual functional behaviour of the standing wave modulation term w_{ssb} needs to be modelled from the OFF measurement.

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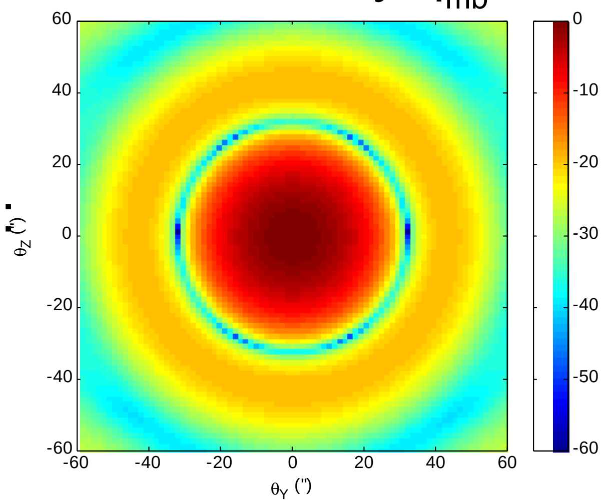
Beam coupling

Determine source coupling factor η_{sf}:

$$T_A^*(\theta,\phi) = \frac{1}{\eta_l \Omega_A} \int_{source} P(\theta-\theta',\phi-\phi') J_\nu(T_B) \psi(\theta',\phi') d\Omega'$$

- Two different efficiencies relevant:
  - Point sources: aperture efficiency η_A
  - Extended sources (comparable to beam): main beam efficiency η_{mb}
    - Part of the beam within the first zero
    - Problem: no good zero for real telescope
  - But both efficiencies represent idealized cases:
    - There are no good calibrators to measure them directly!



Aperture and main beam efficiency are coupled through the main beam size:

$$\eta_{mb} = \eta_A \frac{A_{geom}\Omega_{mb}}{\lambda^2}$$

Beam coupling

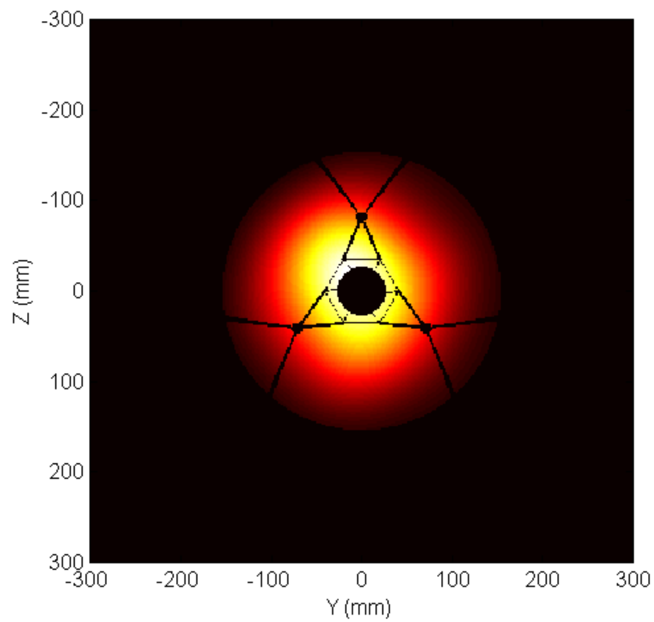
Computation for Gaussian beam:

$$P(\theta) = \exp[-\ln 2 (2\theta/\theta_b)^2]$$

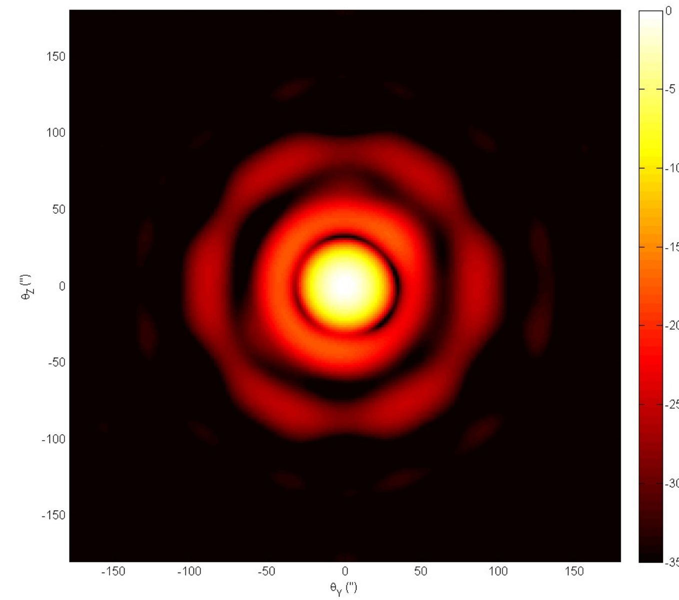
- We only have to measure 2 quantities out of η_A, η_{mb}, Θ_{mb}

$$\Omega_{\mathrm{mb}} = \frac{1}{4\ln 2}\pi\theta_b^2 \approx 1.133\,\theta_b^2$$

But: real beam non-Gaussian: (see David's talk)



Theoretical illumination pattern



HIFI beam still not measured to level below -17dB

Line calibration

Separation of lines and continuum:

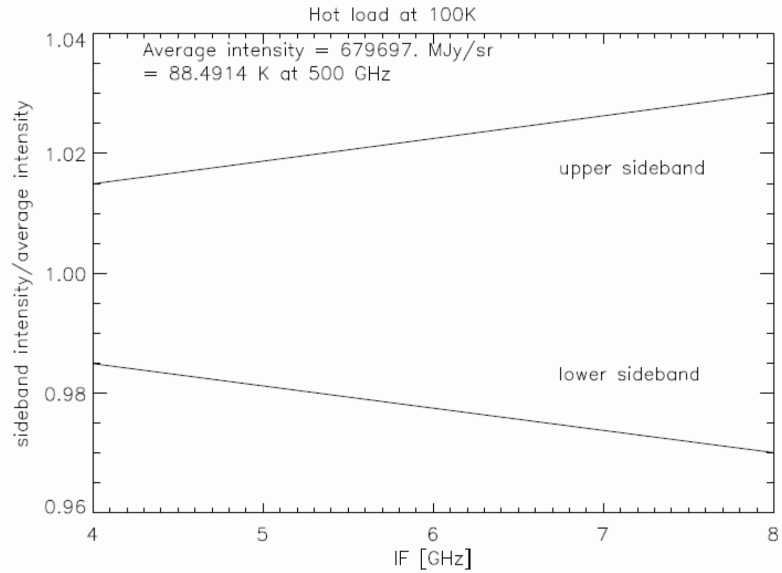
To determine the continuum contribution, the baseline has to be fitted by

$$J_{\text{source,cont}} - J_{\text{OFF,cont}} = J_{\text{source,LO}}\left[1 + R \pm b_{\text{source}}\nu_{\text{IF}}(1-R)\right] - J_{\text{OFF,LO}}\left[1 + R \pm b_{\text{OFF}}\nu_{\text{IF}}(1-R)\right]$$

with the channel dependent sideband ratio

$$R = \frac{\eta_{\text{l}}(1-G_{\text{ssb}}) + w_{\text{isb}}}{\eta_{\text{l}} G_{\text{ssb}} + w_{\text{ssb}}}$$

The continuum level of the astronomical source is then given by the amplitude $J_{\text{source,LO}}$ at the LC frequency and its spectral slope b_{source}.



Reality:

- missing model for sideband specific standing waves
- Low accuracy of sideband gain determination (see David's talk)

Line calibration

Resulting line intensity:

$$J_{\mathrm{S,lines}} - J_{\mathrm{R,lines}} = \frac{1}{G_{\mathrm{ssb}}}\left[\frac{c_{\mathrm{S}} - c_{\mathrm{R}}}{\eta_{\mathrm{S}}\eta_{\mathrm{l}}\gamma^{\mathrm{l}}_{\mathrm{rec}}} - (J_{\mathrm{S,LO}} - J_{\mathrm{R,LO}}) \mp (2G_{\mathrm{ssb}} - 1)\,(J_{\mathrm{S,LO}}b_{\mathrm{S}} - J_{\mathrm{R,LO}}b_{\mathrm{R}})\,\nu_{\mathrm{IF}}\right]$$

Standing waves "hidden" in G_{ssb} , γ_{rec} and η_l.

Example:

Standing wave changing the gain

	Total power	Chop	Load chop	Frequency switch
γ_{ssb}	$\gamma^{\mathrm{l}}_{\mathrm{rec}}G_{\mathrm{ssb}} + w_{\mathrm{ssb}}$	$\gamma^{\mathrm{l}}_{\mathrm{rec}}G_{\mathrm{ssb}} + w^{\mathrm{ON}}_{\mathrm{ssb}}$	$\gamma^{\mathrm{l}}_{\mathrm{rec}}G_{\mathrm{ssb}} + w_{\mathrm{ssb}}$	$\gamma^{\mathrm{l,ON}}_{\mathrm{rec}}G_{\mathrm{ssb}} + w^{\mathrm{ON}}_{\mathrm{ssb}}$
γ_{isb}	$\gamma^{\mathrm{l}}_{\mathrm{rec}}(1 - G_{\mathrm{ssb}}) + w_{\mathrm{isb}}$	$\gamma^{\mathrm{l}}_{\mathrm{rec}}(1 - G_{\mathrm{ssb}}) + w^{\mathrm{ON}}_{\mathrm{isb}}$	$\gamma^{\mathrm{l}}_{\mathrm{rec}}(1 - G_{\mathrm{ssb}}) + w_{\mathrm{isb}}$	$\gamma^{\mathrm{l,ON}}_{\mathrm{rec}}(1 - G_{\mathrm{ssb}}) + w^{\mathrm{ON}}_{\mathrm{isb}}$
$\eta_{\mathrm{l,ssb}}$	η_{l}	η_{l}	η_{l}	η_{l}
$\eta_{\mathrm{l,isb}}$	η_{l}	η_{l}	η_{l}	η_{l}
$\eta_{\mathrm{S,ssb}}$	η_{S}	η_{S}	η_{S}	η_{S}
$\eta_{\mathrm{S,isb}}$	η_{S}	η_{S}	η_{S}	η_{S}
γ_{rec}	$\gamma^{\mathrm{l}}_{\mathrm{rec}}$	$\gamma^{\mathrm{l}}_{\mathrm{rec}}$	$\gamma^{\mathrm{l}}_{\mathrm{rec}}$	$\gamma^{\mathrm{l,ON}}_{\mathrm{rec}}$
J_{rec}	$J^{\mathrm{l}}_{\mathrm{rec}}$	$J^{\mathrm{l}}_{\mathrm{rec}}$	$J^{\mathrm{l}}_{\mathrm{rec}}$	$J^{\mathrm{l,ON}}_{\mathrm{rec}}$

Calibration accuracy

Composed error:

$$\frac{\delta(J_{\mathrm{S,lines}} - J_{\mathrm{R,lines}})}{J_{\mathrm{S,lines}} - J_{\mathrm{R,lines}}} \approx \sqrt{\frac{\delta c_{\mathrm{S}}^2 + \delta c_{\mathrm{R}}^2}{(c_{\mathrm{S}} - c_{\mathrm{R}})^2} + \frac{\delta \gamma_{\mathrm{rec}}^{\mathrm{l}}{}^2}{\gamma_{\mathrm{rec}}^{\mathrm{l}}{}^2} + \frac{\delta \eta_{\mathrm{l}}^2}{\eta_{\mathrm{l}}^2} + 2\frac{\delta J_{\mathrm{sw}}^2}{J_{\mathrm{sw}}^2}\left(\frac{J_{\mathrm{T,pick}}}{J_{\mathrm{S,lines}} - J_{\mathrm{R,lines}}}\right)^2}$$

= relative error, i.e. accuracy ~ source signal

Radiometric error from bandpass calibration:

$$\frac{\delta \gamma_{\mathrm{rec}}^{\mathrm{l}}}{\gamma_{\mathrm{rec}}^{\mathrm{l}}} \approx \frac{1}{\sqrt{\Delta\nu t_{\mathrm{load}}}} \begin{cases} 2.32 & \text{at } 500\,\mathrm{GHz} \\ 32.7 & \text{at } 1.9\,\mathrm{THz} \end{cases}$$

Integration times necessary to guarantee a calibration error $< 1\,\%$:

$\delta\nu$	500 GHz	1.9 THz
1 MHz	0.1 s	10.7 s
0.14 MHz	0.4 s	76.4 s

From OFF calibration:

$$\frac{\delta J_{\mathrm{sw}}}{J_{\mathrm{sw}}} = \begin{cases} \sqrt{\frac{61^2}{\Delta\nu t_{\mathrm{OFF}}} + \frac{68^2}{\Delta\nu t_{\mathrm{load}}}} & \text{at } 500\,\mathrm{GHz} \\ \sqrt{\frac{960^2}{\Delta\nu t_{\mathrm{OFF}}} + \frac{950^2}{\Delta\nu t_{\mathrm{load}}}} & \text{at } 1.9\,\mathrm{THz} \end{cases}$$

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Calibration accuracy

Error determined by:

- Integration time on loads
- Integration time on OFF
- Goal frequency resolution of astronomical observation
- Frequency resolution required to represent standing waves
- Source intensity

All integration times are adjusted to reach 1% accuracy but:

- SW period unknown
    - OFF integration times only approximately o.k.
- Strong source continuum affects line accuracy
    - Weak lines are hard to detect on strong continuum
    - No HSPOT user input for source intensity
        - Seemed too complicated

Frequency calibration

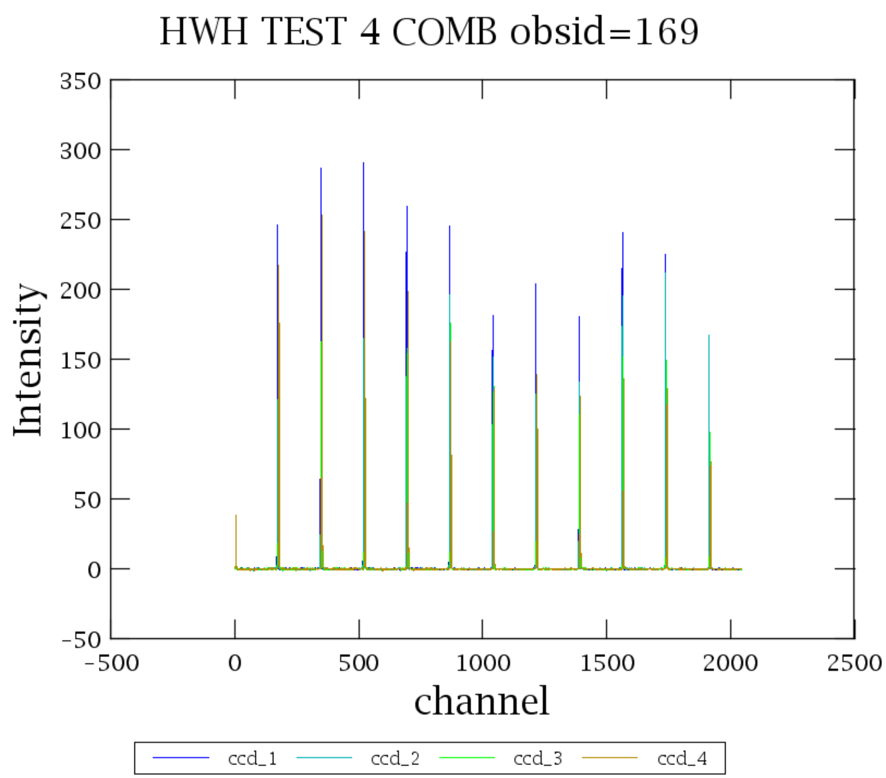
Basic principles:

- frequency scale described by nonlinear “frequency model”
- each channel has an individual frequency assigned
- only optional resampling to a common linear scale

1. Assign IF frequencies

HRS
ACF windowing Symmetrisation Fast Fourier transform IF frequencies from HRS LO settings

WBS
extract comb spectrum measure comb line distance and shape frequency model for each subband



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Calibration accuracy

Accuracy:

- 10MHz master oscillator:
  - Temperature controlled to 1e-8 → 20kHz @ 2THz
- The various LOs and clocks are locked to the 10MHz
- Exception:
  - HEB IF upconvertr free-running 10.4 GHz DRO:
    - Stabilized to <1ppm/degC → ~10kHz (1e-6)

→ onboard frequency uncertain to ~30kHz

- Spectral Resolution: WBS~1.1MHz, HRS~125kHz
- Relative accuracy:
  - WBS within 100kHz (dominated by COMB solutions)
  - HRS within ~50kHz

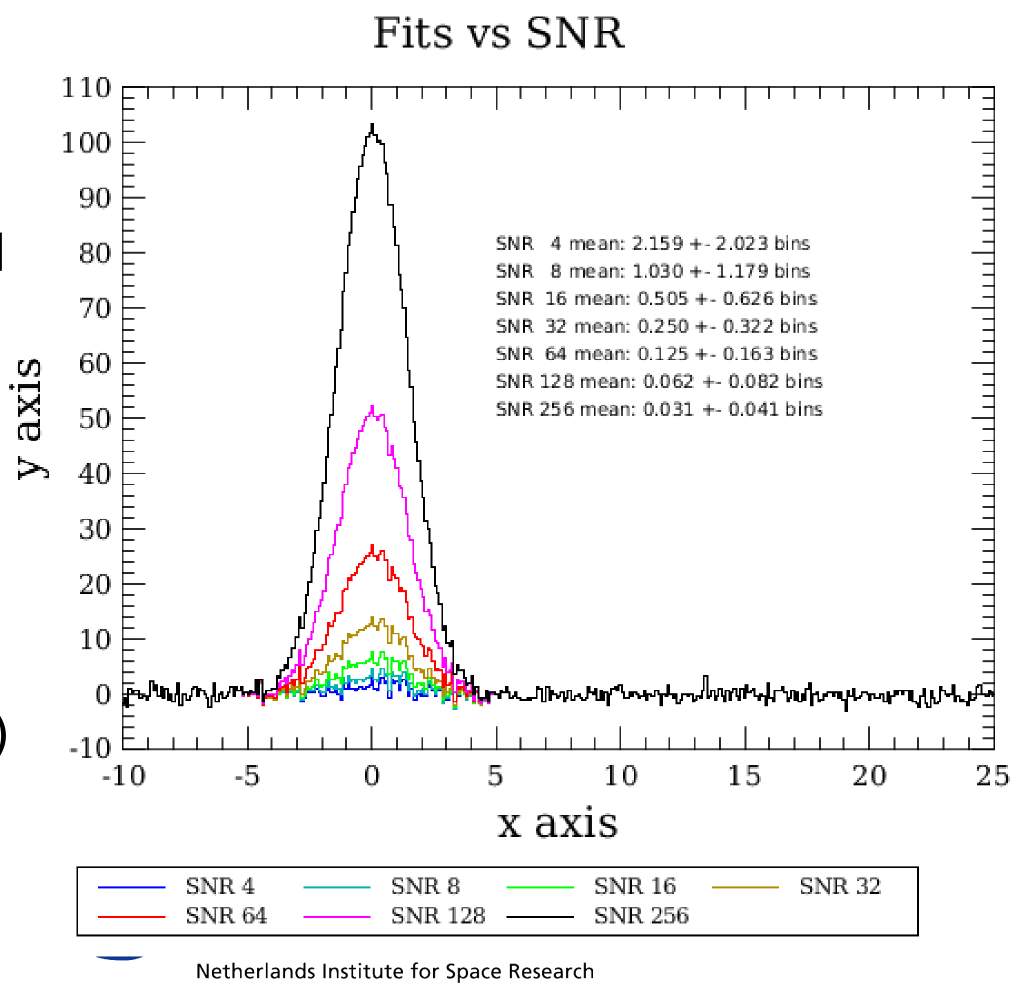
Calibration accuracy

Accuracy:

- Depends on S/N of measurement
  - proportional to 1/SNR
- One can measure moments of a spectral line to higher than instrumental resolution.
  - Applies to comb lines and bright astronomical lines
  - < 100kHz possible

- Problem: Dying hardware of WBS-V comb generator

  → HRS cross calibration scheme (see David's talk)



Sideband assignment

2. Translate to sky frequencies

$$\nu_{usb} = \nu_{LO} + \nu_{IF} \quad ; \qquad \nu_{lsb} = \nu_{LO} - \nu_{IF}$$

- From a single LO measurement it is impossible to find out whether some radiation comes from the LSB or the USB
  - Solution possible for spectral scans combining multiple LO frequencies
- Intensity calibration may depend on sideband
  - For cases with $G_{ssb} \neq 0.5$
- In “impure” frequency regions, HIFI is a multi-sideband instrument
  - Spurious response
  - Sideband gains still not fully known
  - Marked as “spur regions”

Velocity calibration

3. Correct for LOS velocity

$$\nu_{\text{rest}} = \nu_{\text{sky}} \left(1 - \frac{v_{\text{rest,los}} - v_{\text{satellite,los}}}{c}\right)$$

Radio convention (simplified)

- Scaling by the redshift to the frequencies in the defined rest frame.
- Default rest frame:
  - LSR for non-SSOs
  - source frame for SSOs
- HSO motion wrt the Solar System Barycenter measured to precision < 1m/s (1.7kHz @ 500GHz) via the DE405 ephemeris.
- The Solar Local Standard of Rest is defined by a constant velocity offset from the SSBC.
  - 20.0 km/s towards 18h03m50.29s, +30d00'16.8" J2000

Frequency to inertial frame

Magnitude estimates:

- HSO velocity wrt SSBC ~ 30km/s

$$\rightarrow \quad \frac{\Delta\nu}{\nu} \approx -\frac{v}{c} + \frac{1}{2}\left(\frac{v}{c}\right)^2 - \frac{1}{2}\left(\frac{v}{c}\right)^3$$

$$\approx 10^{-4} \qquad \approx 10^{-8} \qquad \approx 10^{-12}$$

→ Full Lorentz transform to the frame of the SSO at time of photon emission (the 'retarded time') required.

- Gravitational redshift of the HIFI clock between aphelion & perihelion

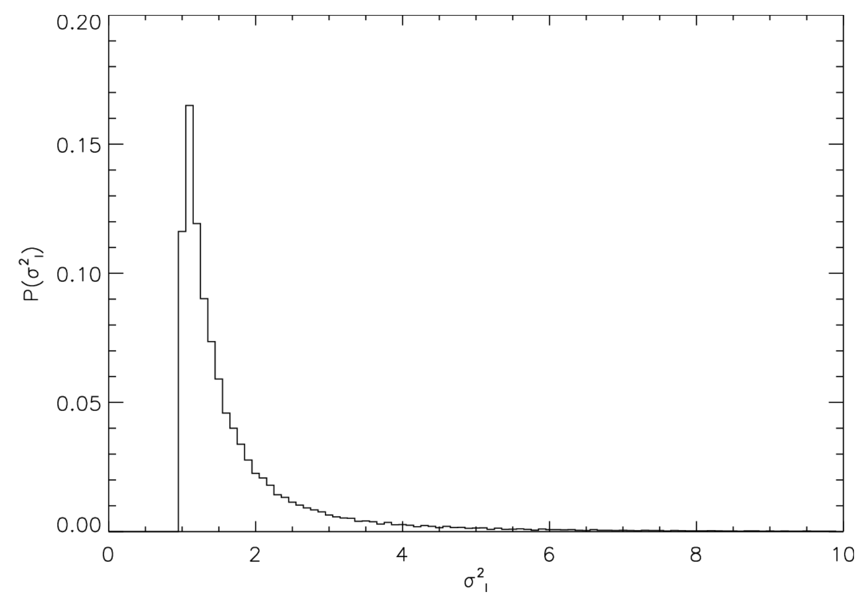
$$\rightarrow \quad \frac{\Delta\nu}{\nu} \approx 2\frac{GM_\odot}{c^2}\frac{1}{a}\frac{e}{1-e^2}$$

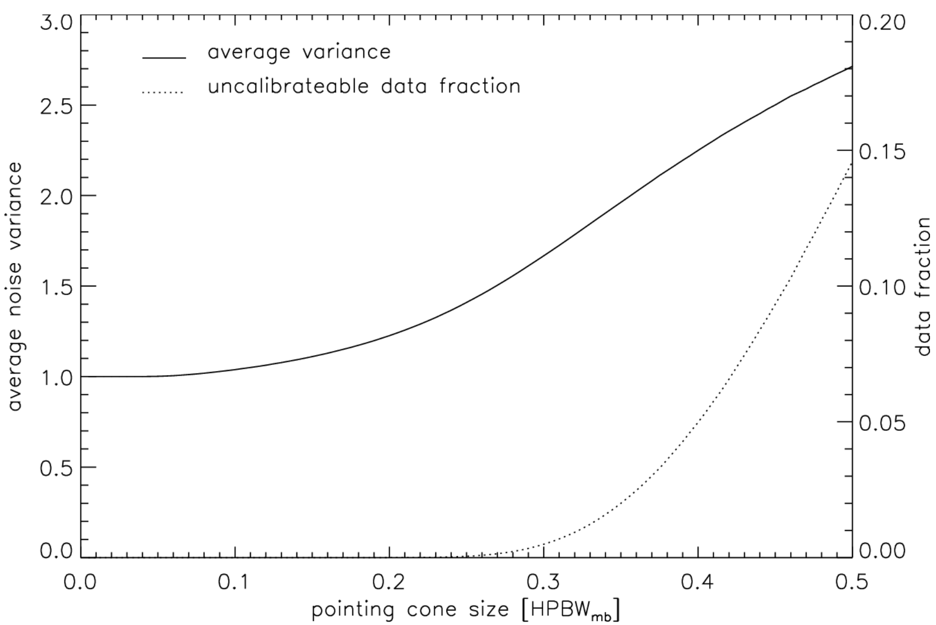
$$\approx 3 \times 10^{-10}$$

→ GR corrections not necessary for HIFI

Pointing calibration

- Probability distribution of calibration uncertainty enhancement due to pointing errors in band 7 assuming the baseline 3.7" pointing error cone.

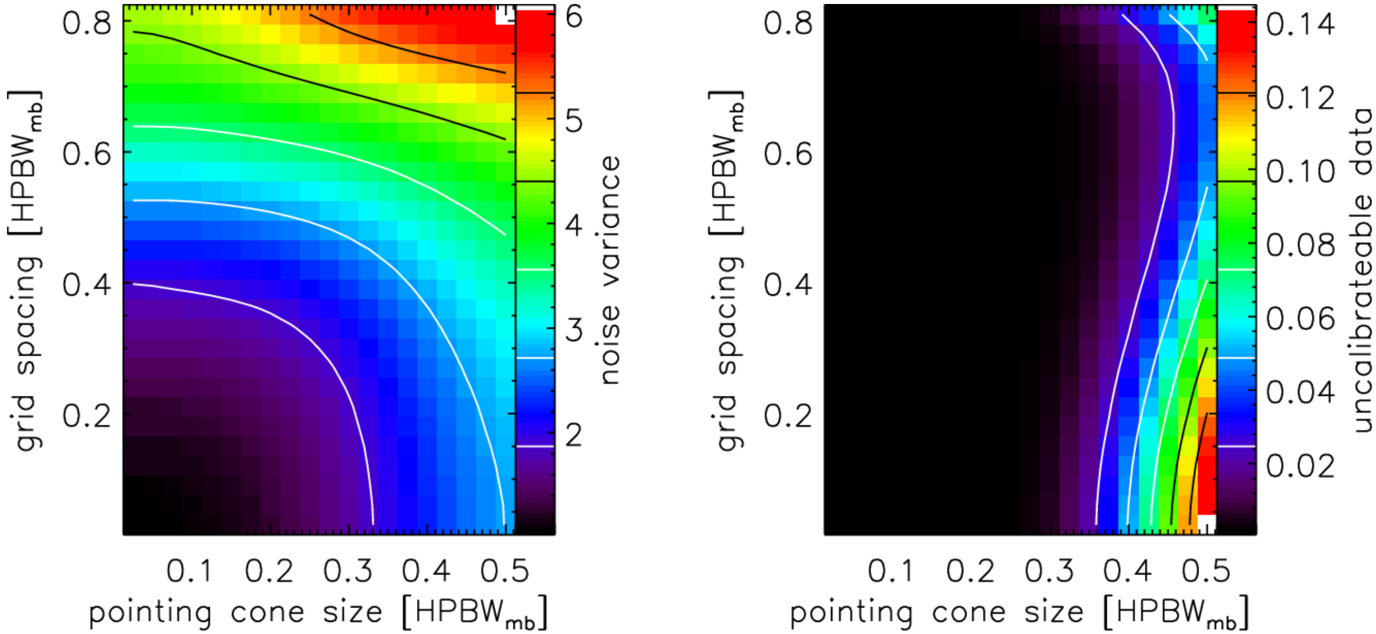




Resulting average data uncertainty due to pointing jitter and fraction of uncalibrateable data (uncertainty increased by > factor 10) as a function of the size of the error cone

Mitigation technique

- Perform 5-point cross maps
  - Uncertainty enhancement compared to perfect staring observation



  - Optimum for “jitter” spacing, i.e. step size = 1 σ pointing error cone
  - Alternative: A-posteriori pointing info from gyro-propagation
- Nothing of that finally used

Summary

All 4 dimensions important:

- Pointing calibration not needed due to improved telescope performance
- Frequency calibration very good after > 1 year of debugging details
- Intensity calibration is still challenging
  - Load calibration very reliable
  - HIFI beam still not well known at the level below -17dB
  - Intensity calibration framework in principle used, but:
    - Standing waves towards hot and cold load higher than expected
    - Still no model to disentangle SW contributions from both sidebands
      → Standing wave are one/the major source of uncertainties
    - Model for IF standing waves almost ready-to-use
    - Sideband gains not accurately measured/still not understood
      → large calibration uncertainty