Star formation

Protostellar accretion disks
Summary of previous lectures and goal for today

Pre-main-sequence phase
- (almost) final mass reached
- visible in optical (envelope dispersed)
- quasi-static contraction

Protostars
- main accretion phase
- not visible in optical (dust envelope)

Pre-Stellar Dense Core
$T_{\text{bol}} \sim 10-20 \text{ K, } M_{\odot} = 0$
- $10^6 \text{ yr}$

Young Accreting Protostar
$T_{\text{bol}} < 70 \text{ K, } M_{\odot} \ll M_{\text{env}}$
- $< 30,000 \text{ yr}$

Evolved Accreting Protostar
$T_{\text{bol}} \sim 70-650 \text{ K, } M_{\odot} > M_{\text{env}}$
- $\sim 200,000 \text{ yr}$

Classical T Tauri Star
$T_{\text{bol}} \sim 650-2880 \text{ K, } M_{\text{disk}} \sim 0.01 M_{\odot}$
- $\sim 1,000,000 \text{ yr}$

Weak T Tauri Star
$T_{\text{bol}} > 2880 \text{ K, } M_{\text{disk}} < M_{\text{Jupiter}}$
- $\sim 10,000,000 \text{ yr}$

Time
Formation of disks

So far, no magnetic field and no rotation...

Now: Let’s look at rotation! (and magnetic fields)

A consequence of rotation: disks

- disk formation
- disk structure

Environmental impact: jets and outflows

- angular momentum removal
Our solar system

• Planetary orbits are coplanar.
• All planets orbit in the same direction
• Sun mass = 98%, sun angular momentum <2%

→ “Nebular hypothesis”:
(famous from Kant & Laplace, 18th century, but actually first exposed by Emanuel Swedenborg in 1734)

“Planets are formed from a gaseous protoplanetary disk”
Consequence of rotation

Presence of accretion disks is predicted by theory of infall of rotating clouds

In the simplistic case of rotating infall \textit{without magnetic braking}, the angular momentum of any fluid particle is conserved during infall.

- The angular rotation increases with decreasing distance to the central axis
  \[ \Omega = \Omega_0 \left( \frac{r_0}{r} \right)^2 \]
- Centrifugal force \( = m\Omega^2r \sim r^{-3} \)
- Gravitational force \( \sim r^{-2} \)

\( \rightarrow \) rotation becomes important close to the central object
The formation of a disk

3-D Radiation-Hydro simulations of disk formation

Yorke, Bodenheimer & Laughlin 1993
Observations of disks

- Direct
  - Optical (silhouette)
    - Not deeply embedded objects
  - Millimeter/Submillimeter
    - Dust
    - Molecular lines
- Indirect
  - SEDs
- Results: Sizes ~ 100 AU
Protoplanetary Disks
Orion Nebula

β Pic

Disks around Young Stars
PRC99-05b • STScI OPO
C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

Protoplanetary Disks
Orion Nebula

HST - WFPC2

MBM12 3C
Jayawardhana et al. 2002

β Pic

Young Stellar Disks in Infrared

HST - NICMOS

Disks around Young Stars
PRC99-05b • STScI OPO
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β Pic

Protoplanetary Disks
Orion Nebula

HST - WFPC2

MBM12 3C
Jayawardhana et al. 2002
GG Tau – the ring world

**Fig. 6a and b.** Comparison of the a $^{13}$CO J=2–1 line emission at $V = 5.55$ (blue contours), 6.30 (white contours) and 7.05 km s$^{-1}$ (red contours), overlaid on a false colour image of the 1.4 mm continuum emission and b J-band emission resolved by RRNGJ. The white ellipse indicates the ring average radius.
Disk evaporation: proplyds

Orion 114-426

McCaughrean & O'Dell 1996
SEDs
Magnetic braking

- Existence of an azimuthal component of $B$.
- Field line twisting due to rotation ($B$ is anchored on large scales).
- This creates a torque which acts against the rotation.
  - Transport of angular momentum by torsional Alfvén waves
- Close to the center, $B$ and matter decouple because density increases and thus ionization decreases.
- $B$ is not anymore “frozen”.
- Magnetic braking only efficient in outer disk.
Consequence of rotation

Matter that falls towards the center “misses” the center because of the centrifugal forces.

If a particle with specific angular momentum $l$ (=angular momentum per unit mass) falls in and ends up on a circular orbit while maintaining its angular momentum, then the radius $r$ of the orbit is:

$$r = \frac{l^2}{GM}$$

In the inside-out infall model, all material arriving at the center at a certain time started from the same radius $r_0$.

If the cloud core is initially in uniform rotation $\Omega$, then the specific angular momentum at $r_0$ varies with angle $\theta$ from the rotation axis:

$$l = \Omega r_0^2 \sin \theta$$
Consequence of rotation

Infalling material from different directions will arrive at the midplane at different radii.

Maximum radius “centrifugal radius” (for $\sin \theta = 1$)

$$r_{\text{cen}} = r_0^4 \frac{\Omega^2}{GM}$$

→ scale of the protostellar disk

Fluid stream lines (solid curves) and isodensity contours (dashed curves) within a collapsing rotating cloud.
Orbits

- Mass does not reach Keplerian perihel, but closes radius where it hits the disk

**Figure 10.15** Parabolic orbit in rotating infall. A fluid element, with instantaneous polar coordinates \((r, \psi)\) in the orbital plane, falls into the radial distance \(r_{eq}\), where it impacts the disk. If the disk were absent, the element would have reached the smaller distance \(r_{\min}\) before swinging back out.
Rotating infall

Axisymmetry + equatorial symmetry

• material falling from above and below the equatorial plan meet on the equator through a shock which dissipates the vertical kinetic energy.

• If cooling efficient: material accumulates and forms a disk

Most of the matter will continue towards the center, whereas some angular momentum is transported outwards by viscosity.

Typical sizes: >100 AU
Streamlines in disks

- The accretion shock occurs on the surface of the disk as well as on the protostar
- Radial infall in the disk
The angular momentum problem

- Angular momentum of 1 M⊙ in 10 AU disk: $3 \times 10^{49}$ m²/s
- Angular momentum of 1 M⊙ in 1 R⊙ star: $<< 6 \times 10^{47}$ m²/s (=breakup-rotation-speed)
- Original angular momentum of disk = 50x higher than maximum allowed for a star
- Angular momentum is strictly conserved!
- Two possible solutions:
  - Torque against external medium (via magnetic fields?)
  - Very outer disk absorbs all angular momentum by moving outward, while rest moves inward.
Need *friction through viscosity!*
Angular momentum transport

- Ring A moves faster than ring B.
- Friction between the two will try to slow down A and speed up B.
- Angular momentum is transferred from A to B.

Specific angular momentum for a Keplerian disk:

\[ l = rv_f = r^2 \Omega_K = \sqrt{GM} r \]

So if ring A loses angular momentum, but is forced to remain on a Kepler orbit, it must move inward! Ring B moves outward, unless it, too, has friction (with a ring C, which has friction with D, etc.).
Formation & spreading of disk
Formation & spreading of disk
Formation & spreading of disk
Non-stationary (spreading) disks

\[ r_{\text{cen}} \approx \frac{c_s \Omega_0^2 t^3}{16} = 0.3 \, AU \left( \frac{T}{10 \, K} \right)^{1/2} \left( \frac{\Omega_0}{10^{-14} \, s^{-1}} \right)^2 \left( \frac{t}{10^5 \, yrs} \right)^3 \]

Lynden-Bell & Pringle (1974), Hartmann et al. (1998)
Birth of the disk

Critical time determined by $r_{\text{cen}} = R_*$

$$t_0 = \left( \frac{16 R_*}{\Omega_0^2 v_s} \right)^{1/3} = 3 \times 10^4 \text{[yr]} \left( \frac{R_*}{3\odot} \right)^{1/3} \left( \frac{\Omega_0}{10^{-14} \text{s}^{-1}} \right)^{-2/3} \left( \frac{v_s}{0.3 \text{ km s}^{-1}} \right)^{-1/3}$$

the mass of the protostar is $M_* = \dot{M} t_0$

leading to initial stellar mass:

$$M_0 = \left( \frac{16 R_* v_s^8}{G^3 \Omega_0^2} \right)^{1/3} = 0.2 \odot \left( \frac{R_*}{3\odot} \right)^{1/3} \left( \frac{\Omega_0}{10^{-14} \text{s}^{-1}} \right)^{-2/3} \left( \frac{v_s}{0.3 \text{ km s}^{-1}} \right)^{8/3}$$
Disk evolution

- Viscous angular momentum transport in a fully evolved accretion disk enables efficient accretion
- Accretion fades with outer supply
  → birth of PMS

Fig. 1.— Evolution of various disk and star quantities as a function of time after the onset of collapse of the cloud core (after Hueso and Guillot, 2005). Solid lines: stellar mass (upper) and disk mass (lower). Dotted line: accretion rate in the disk. In this model the disk is formed at $t \approx 0.03$ Myr, causing the jump in the dotted line at this point. The collapse phase is finished by $2 \times 10^5$ years.

Hueso & Guillot (2005)
Disk evolution

- Spiral patterns expands with $t^3$
- At time $t_1 = 1.43 \ t_0$, the streamlines start missing the protostar (at a radius of $0.34 \ r_{\text{cen}}$)
- boundary between (tenuous) outer disk and (dense) inner disk with almost circular orbits

Figure 11.12 Early expansion of a protostellar disk. (a) Before time $t_1$, curved streamlines from the outer disk impact the protostar directly. (b) After $t_1$, these streamlines converge to form a dense ring, which transfers mass to an inner disk surrounding the star. At all times, the outer disk boundary is the growing centrifugal radius $\omega_{\text{cen}}$. 
Mass transport rate:

- Standard 2-D disk models provide a constant accretion rate across the disk
- Spirals lead to spatial and temporal discontinuities

**Fig. 11.**—Mass transport rate in the disk. Displayed are both $\dot{m}_{\text{inner}}$ and $\dot{m}_{\text{outer}}$, as derived in the text. Note the discontinuity in the transport rate at $u = u_{\text{crit}}$, where the ring is being built up. As in Fig. 10, the true inner boundary is at $u = u_*$. 
Accretion in an inhomogeneous disk

Temporal evolution of mass accretion rate:

FU Orionis outbursts:
- from spiral inflow channel
- Clumpy disk

→ still not fully understood
FU Orionis outburst

- Mass accretion rates varies from $10^{-7} \, M_\odot$/yr to $10^{-4} \, M_\odot$/yr
- Origin: mass pileup in disk – relation to spirals unclear
- Thermal or flow instabilites probably trigger outbursts

Figure 1. Light curves of V1057 Cyg (left), V1515 Cyg (middle) and FU Ori (right). Squares indicate our new data, while triangles indicate historical data taken from Mendoza (1971), Rieke, Lee & Coyne (1972), Schwartz & Snow (1972), Bossen (1972), Welin (1975, 1976), Landolt (1975, 1977), Kolotilov (1977), Kopatskaya (1984) and Ibragimov & Shevchenko (1988).
Disk evolution

Disk dispersal:

Observations of IR excess show that the disk lifetime is only a few million years.

Sets the time scale for
- Angular momentum transfer
- Planet formation

Haisch et al. 2000
Angular momentum transfer

Angular momentum:
Accretion from the Keplerian disk adds angular momentum to the pre-main sequence star.

It takes $10^8$ a to remove the angular momentum by stellar radiation.

→ Young stars rotate much faster than main sequence stars.

Fig. 37. The evolution of the angular velocity $\Omega$ for a 1 $M_\odot$ star as a function of time for various disk life times. The uppermost curve is for stars with short lived disks ($4 \times 10^5$ yr), while the lowest one is for long-lived disks ($30 \times 10^6$ yr). The observed angular velocities in a variety of T Tauri stars and open clusters are shown by the various symbols. (From Bouvier et al. [10])

Typical TTau rotation at 1/10 of disruption velocity
The density structure

Consider density scaling in inner main part:

- Self-similar collapse in 2-D would provide $\sigma \sim r^{-1.5}$
- Viscous disk:
  - Density structure determined by flow of material:
  - Assumess constant flow of material through the disk (continuity)
  - Viscosity $\nu$ determines mass and angular momentum transfer

\[ \sigma = \frac{\dot{M}}{3\pi \nu} \]
The disk viscosity

Molecular viscosity: \[ v = \langle v_T \rangle \lambda_{\text{free}} \approx c_s \lambda_{\text{free}} \]

- \( \lambda_{\text{free}} \) = mean free path of molecule

Typical disk (at 1 AU): \[ N = 1 \times 10^{14} \text{ cm}^{-3}, \ T = 500 \text{ K}, \ L = 0.01 \text{AU} \]
\[ \sigma_{H_2} \approx \pi (1 \text{Å})^2 \rightarrow \lambda_{\text{free}} = 1/(N \sigma) \approx 32 \text{cm} \]
\[ \rightarrow \quad v = 730 \text{ m}^2/\text{s} \]

- Extremely low
- Would allow a maximum accretion rate of \( 3 \times 10^{-12} \ M_\odot/\text{a} \) for a \( 1M_\odot \), 100AU disk
- Actual viscosity must be > \( 10^6 \) times higher
  \( \rightarrow \) provided by turbulent, not thermal motion
The disk viscosity

Turbulent viscosity: Shakura-Sunyaev model

- Dissipation provided at scale of minimum eddy size
- Characteristics: scale height $h$, sound speed $c_s$
- Additional “efficiency” factor $\alpha$

\[ \nu = \alpha c_s h \quad \alpha = 0.001 \ldots 0.1 \]

- Scale height of disk:
  - Solution of hydrostatic balance in z-direction
  - \[ \rho(z) = \rho_0 \exp \left( -\frac{z^2}{2h^2} \right) \] with \[ h = \sqrt{\frac{kT r^3}{\mu m_p GM_x}} = \frac{c_s^2}{\Omega_{\text{Kepler}}} \]

A Gaussian!
The density structure

Disk height: \( \Omega_{\text{Kepler}} \propto r^{-3/2} \)

\[ h \propto r^{3/2} \quad \text{for constant temperature} \]

\[ \rightarrow \text{“Flared disk”} \]

Flared disk with inner rim from heating

- Resulting surface density

\[ \sigma = \frac{\dot{M}}{3\pi \nu} \propto r^{-3/2} \]

- However, this ignores all heating processes by assuming constant \( c_s \).
The density structure

Temperature structure
→ disk height
→ density structure:
The temperature structure

2 Heating processes:

• Viscous heating from the friction within the disk
  • Only relevant for massive, optically thick disks
• Irradiation of the surface from the protostar/PMS

Viscous heating:

• Dissipation in shear flow:

\[
\frac{\partial \Omega}{\partial r} = -\frac{3}{2} \sqrt{\frac{GM_*}{r^{5/2}}}
\]

• Keplerian orbits:

\[
\dot{E} = -2\sigma_S T^4
\]

• Radiative cooling in optically thick disk:

\[
T = \left( \frac{9GM_*\nu\sigma}{8\sigma_S r^3} \right)^{1/4} = \left( \frac{3GM_*\dot{M}}{8\pi\sigma_S r^3} \right)^{1/4} \propto r^{-3/4}
\]
Radiative heating

Irradiation from the central star heats gas & dust in disks

- $T \sim r^{-1/2}$ for optically thick disk
- $T \sim r^{-2/(4+\beta)}$ for optically thin disk with

$$\dot{E} = -2\sigma_S T^{4+\beta} \frac{Q_0 \sigma}{2}$$
Temperature Structure

Verification by observations:

- Observable spectrum from unresolved disk
  - Determine spectral index $\nu S_\nu(\nu) \propto \nu^\alpha$
  - Most T Tau disks: $\alpha \approx 0$

- Theoretical spectrum of optically thick disk:

  $$S_\nu(\nu) = \int_{R_{in}}^{R_{out}} \frac{2h\nu^3}{c^2 \left[ \exp \left(\frac{h\nu}{kT}\right) - 1 \right]} \frac{2\pi R \, dR}{D}$$

  - With $T \sim r^{-p}$

  $$\rightarrow \nu S_\nu(\nu) \propto \nu^{4-2/p}$$

  - Most disks must have $p = \frac{1}{2}$

- Contribution of viscous heating negligible

- Support of Flared disk model with large surface
Disk SED

Fit of the spectral energy distribution in the Rayleigh-Jeans domain

- Dominated by central layer with high density
- Optically thick emission

Fig. 2.— Build-up of the SED of a flaring protoplanetary disk and the origin of various components: the near-infrared bump comes from the inner rim, the infrared dust features from the warm surface layer, and the underlying continuum from the deeper (cooler) disk regions. Typically the near- and mid-infrared emission comes from small radii, while the far-infrared comes from the outer disk regions. The (sub-)millimeter emission mostly comes from the midplane of the outer disk. Scattering is not included here.

Dullemond et al. 2006 PPV
Disk SED

Flared disk with inner rim:
- Temperature increase at inner boundary produces additional pressure
- Inner rim is “puffed-up”
- Shadowed region behind rim

Fig. 5.— Overall SED shape for non-accreting disks with stellar irradiation, computed using the 2-D radiative transfer tools from Dullemont & Dominik (2004a). The stellar spectrum is added in grey-scale. Scattered light is not included in these SEDs. Solid line is normal flaring disk with inner dust rim; dashed line is when the rim is made higher; dot-dashed line is when the flaring is reduced (or when the disk becomes ‘self-shadowed’); dotted line is when the inner rim is at 10× larger radius.
Temperature Structure

Strong temperature gradient in $z$ from irradiation:

Surface temperature determined by local gas cooling

Temperature of central layer dominated by overall gas density and accretion flow

Fig. 4.— Vertical temperature distribution of an irradiated $\alpha$-disk at 1 AU, for a fixed $\Sigma$ (chosen to be that of a disk model with $\dot{M} = 10^{-8} M_\odot/\text{yr}$ for $\alpha = 0.01$), but varying $\dot{M}$, computed using the models of D'Alessio et al. (1998). The labels of the curves denote the 10-log of the accretion rate in $M_\odot/\text{yr}$. 
Chemical Structure of PPDs

Surface layer: $n \approx 10^{4-5} \text{cm}^{-3}$, $T > 50K$

Photochemistry

Intermediate: $n \approx 10^{6-7} \text{cm}^{-3}$, $T > 40K$

Dense cloud chemistry

Midplane: $n > 10^7 \text{cm}^{-3}$, $T < 20K$

Freeze-out
Fig. 2. Vertical distributions at $R = 373$ AU of a) temperature, b) density ($n_H \equiv 2n(H_2) + n(H)$), c) attenuation of the interstellar radiation ($A_v^\text{IS}$) and stellar radiation ($A_v^\text{star}$), d–e) molecular abundances in the D'Alessio et al. model, and f) molecular abundances in the Kyoto model with $S = 0.03$. In panels a–c), the physical parameters of the D'Alessio et al. and Kyoto models are shown via solid and dashed lines, respectively. In panels d–f), the disk age is assumed to be $t = 1.0 \times 10^6$ yr.
Structure of disks

Fig. 13.— Pictograms of the structure of a flaring protoplanetary disk, in dust (left) and gas (right).
Photoevaporation

- Dust evaporates close to the central star
- Material still flows through the gap onto the central object

- Evaporation from the surface of the disk into a disk wind/outflow if

$$c_s > v_{esc} \approx \left( \frac{GM_*}{r} \right)^{1/2}$$

![Graph showing evolution of surface density of a EUV-photoevaporating disk.](image)

Fig. 11.— Evolution of the surface density of a EUV-photoevaporating disk (Figure adapted from Alexander et al., 2006b). This simulation starts from a given disk structure of about 0.05 $M_\odot$ (marked with ‘Start’ in the figure). Initially the disk accretes and viscously spreads (solid lines). At $t = 6 \times 10^6$ yr the photoevaporation starts affecting the disk. Once the EUV-photoevaporation has drilled a gap in the disk at $\sim 1$ AU, the destruction of the disk goes very rapidly (dashed lines). The inner disk quickly accretes onto the star, followed by a rapid erosion of the outer disk from inside out. In this model the disk viscosity spreads to $> 1000$ AU; however, FUV-photoevaporation (not included) will likely truncate the outer disk.

Dullemond et al. 2006 PPV
Disk wind

Figure 1  Schematic picture of FU Ori objects. FU Ori outbursts are caused by disk accretion increasing from $\sim 10^{-7} \, M_\odot \, \text{yr}^{-1}$ to $\sim 10^{-3} \, M_\odot \, \text{yr}^{-1}$, adding $\sim 10^{-2} \, M_\odot$ to the central T Tauri star during the event. Mass is fed into the disk by the remnant collapsing protostellar envelope with an infall rate $\lesssim 10^{-5} \, M_\odot \, \text{yr}^{-1}$; the disk ejects roughly 10% of the accreted material in a high-velocity wind.

→ Leads us to outflows
Scenario for star- and planet formation

**Single isolated low-mass star**

- $n \sim 10^4 - 10^5 \text{ cm}^{-3}$
- $T \sim 10 \text{ K}$

Cloud collapse

- $t = 0$
- Factor 1000 smaller

Protostar with disk

- $n \sim 10^5 - 10^8 \text{ cm}^{-3}$
- $T \sim 10 - 300 \text{ K}$
- $t = 10^5 \text{ yr}$

Formation planets

- $t = 10^6 - 10^7 \text{ yr}$

Solar system

- $t > 10^8 \text{ yr}$

Only molecules and dust can trace the earliest stages of protostellar evolution