Looking for gravitational lensing signals in the Fermi GRBs

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Abstract. We've analyzed the Fermi high time resolution GRB lightcurves' autocorrelation function looking for gravitational lens effects. The theoretical amplification-time delay function was applied with a photon count S/N approximation to determine the optimal photon binning. We conclude that during the lens signal search among the Fermi data the usual linearity based techniques (e.g. matched filters, FIR filtering, Bayes blocks, autocorrelation function) are sub-optimal. Only the short and very bright GRBs can be the plausible candidates of these detection techniques.

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GRAVITATIONAL LENSING

GRBs are located at high-z therefore the probability of the gravitational lensing by a foreground object is non-negligible. The observed lensed quasar images' separations are smaller than the positional resolution of the current gamma-ray detectors, hence during the search we have to look for the signals imprinted on the lightcurves of the bursts. The time resolution of gamma-ray instruments is superior in many aspects to the optical instruments and this μs resolution photon data can carry the lensing signal. Here we are looking for lensing signal *within* a burst, i.e. we try to find the lensing signal in the lightcurve when there's a lensing object at redshift z_{lens} between the event and us. Here a point mass lens model were used: nevertheless the analysis could be easily extended for more structured models too.

Relevant equations for gravitational lensing can be found in e.g. [1, 2]. The lens' Einstein-radius (in radians) is $\theta_E = \sqrt{2R_s \frac{D_{sl}}{D_s D_l}}$, while from the lensing equation we know that $\beta = \theta - \theta_E^2/\theta$. Here D_s is the luminosity distance between the GRB and the observer, D_{sl} is the luminosity distance between the GRB and the lens, D_l is the luminosity distance between the lens and the observer, $R_s = 2GM_{lens}/c^2$ is the Schwarzschild radius of the lens, θ is the observed angle of the image of the source with respect to the lens and $\beta = f\theta_E$ is the real angle between the source and the lens.

For a point mass lens any point-source will produce exactly two images. Hence the lens' $l(t)$ linear response function (Green's function, convolution function) will consists of two Dirac deltas with a following τ time delay and α relative amplitude/amplification:

$$\tau(f) = \frac{1+z_{lens}}{c} 2R_s \left(\frac{2}{f+\sqrt{f^2+4}} - \frac{2}{f+\sqrt{f^2-4}} + \ln \frac{f+\sqrt{f^2+4}}{f-\sqrt{f^2+4}} \right)$$

$$\alpha(f) = \theta_1/\theta_2 = \frac{f+\sqrt{f^2+4}}{f-\sqrt{f^2+4}}$$

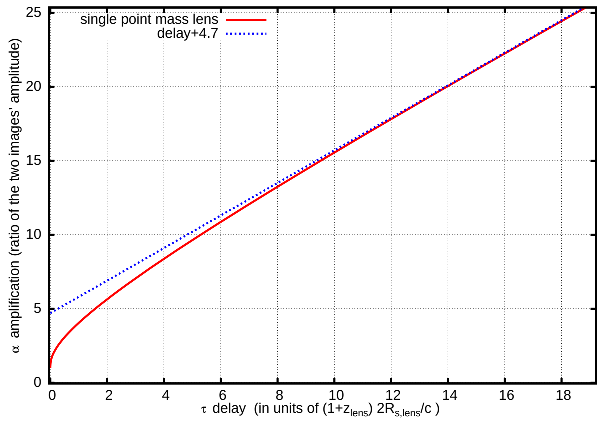

FIGURE 1. The theoretical amplification-delay diagram for a point mass lens approximation, f is the parametric value. For a $\tau > 6$ delay the amplification ratio grows approximately linearly with τ.

FERMI LIGHTCURVES

The Fermi database contains the NaI detectors' GRB event data on a photon-by-photon basis, providing us a powerful tool in the search for a real lensing effect. For our analysis we used all photons between $18-882$keV for each triggered GBM detectors. Each $s(t)$ lightcurve was formed with a time resolution of a given Δ bin width. The question is the optimal Δ value for a lens effect search.

Suppose that we observe a total N_{tot} photons over a time of T. For a typical burst $N_{tot} \approx 10^5$ and $T \approx 100$s, and the average intensity is $n_0 = N/T \approx 1$ photon/ms. Usually the signal and the background contains of a comparable number of photons.

Let us consider a toy model, a very short GRB, appearing only in one Δ bin with D photons. The condition for detection is that the signal should be above the noise, so we can introduce the signal-to-noise ratio of

$$S/N = \frac{D - n_0\Delta}{\sqrt{D + n_0\Delta}}$$

On Fig. 2. the S/N is plotted for different Δ bin widths, with typical N_{tot} values and with an very optimistic $D = 50$.

OPTIMAL BINNING

The $l(t)$ linear response function of the lens should be convolved with the $s(t)$ GRB lightcurve to get the observed data. Denoting the Fourier transform by F, the $R(t)$

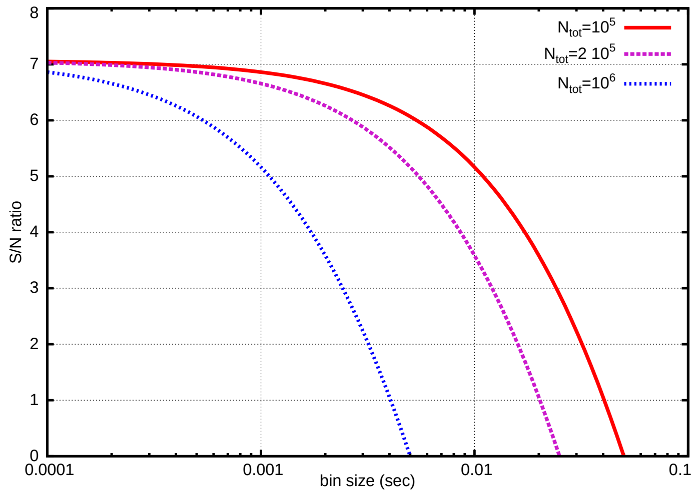

FIGURE 2. S/N values for different bin widths for a typical Fermi GRB.

autocorrelation of the observed signal becomes :

$$\begin{aligned} R(l(t)*s(t)+n(t)) &= R(l(t)*s(t))+R(n(t)) \\ &= F^{-1}(F(l(t)*s(t))^* F(l(t)*s(t)))+F^{-1}(N^2(\omega)) \\ &= F^{-1}(L(\omega)^* S(\omega)^* L(\omega) S(\omega))+F^{-1}(N^2(\omega)) \\ &= F^{-1}(L(\omega)^2 S^2(\omega)+N^2(\omega)) \end{aligned}$$

The optimal binning value would be a filter which optimally filters $(L(\omega)^2 S^2(\omega)$ and supresses the noise. The $l(t)$ consists of two Dirac deltas, therefore $L(\omega)$ will be a $\sin/\cos$ shaped filter, non-vanishing over the whole ω. As a supremum condition, we can assume that $L(\omega)$ is constant: this reduces the optimal binning problem to an optimal Wiener filtering problem, with solution of

$$H(\omega) = \frac{S^2}{S^2+N^2}$$

The typical Fermi GRB's power spectrum extends up to a few Hz, above the signal fades into the noise. Therefore the optimal binning from the S/N point of view gives us a (relatively) large optimal bin size, above the range of a few 10 ms. This values are clearly out of the range of S/N constraints on Fig. 2., hence generally it's no use using linear filtering! Only very short ($\approx$ a few Δ) bursts with a veryhigh peak count could theoretically be used for such filtering during lens searches.

Another observational constraint could be obtained by applying the Fig. 1. relation on the Fig. 2. curves. On Fig. 3. the τ delay is plotted against the Δ bin size for a typical $z_{lens} = 1$ lens with different M_{lens} masses. The ultimate constraint for any technique is the Nyquist-Shannon sampling theorem (the sampling rate is more than twice the signal's maximum frequency): any observation should be above this line.

For the Fermi observations the practical lower limit for Δ is around 1ms, because below this value the detector's dead time and pile up will be a problem. Even our (slightly) optimistic toy model gives a lower limit of $M = 100M_\odot$ for $\Delta \approx 1$ms and $M = 10^6 M_\odot$ for $\Delta \approx 6$ms bin width. These values refine our previous statement: not only the short and bright GRBs are the only plausible candidates but even for a such a burst the measurable lens masses will be limited above $\approx 100M_\odot$.

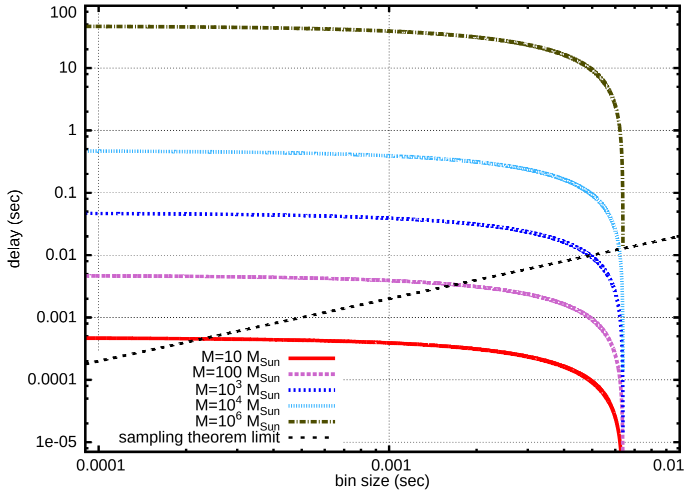

FIGURE 3. The time delay and the bins size diagram for our toy model with $z_{lens} = 1$. Real observations should be above the sampling theorem's limit.

SUMMARY

We've found that linear filtering techniques are sub-optimal during the search for gravitational lens effects in the Fermi GRB data. The possible detectable lens masses will be limited above $\approx 100M_\odot$ for bursts with a typical z and lightcurve. The typical GRBs' autocorrelation function with the point-mass lensing effect requires a high quality (S/N) lightcurve, beyond the realm of current detectors.

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